RESEARCH ARTICLE

2023, vol. 10, issue 1, 284-313
https://doi.org/10.5281/zenodo.8241416

FINANCIAL DIGITAL ASSETS AND THEIR INTERACTIONS WITH THE TRADITIONAL FINANCIAL MARKETS: A DSGE ANALYSIS

Andrei-Dragos Popescu [1]
Cristi Spulbar [2]

[1] PhD Student, Doctoral School of Economic Sciences, University of Craiova, Craiova, Romania
andrei@scx.holdings
[2] Professor, Doctoral School of Economic Sciences, University of Craiova, Craiova, Romania
cristi.spulbar@feea.ucv.ro

Abstract

We start from the premise that all markets, regardless of the shocks they experience, tend toward equilibrium. This is a characteristic that has been identified for any type of market, as the balance is required to be achieved in order for them to evolve. We are evolving a Dynamic General Stochastic Equilibrium model (DSGE) in order to assess and analysis the capital flows of shocks identified within different digital and traditional markets. The model is based on the fundamental theory of general equilibrium which attempts to describe the fluctuations based on supply, demand, and prices in a whole economic scenario, where all markets interact with each other. For our analysis, we have used the historical price data from various financial digital assets and traditional finance markets, with a quarterly time frame from January 2016 to October 2022. The findings show that when a productivity shock occurs, all of the variables respond in a favorable manner, aligning with our utility function and the equations defined in our methodologies. The results reveal that the market is able to adjust and adapt to changes in productivity, leading to an overall improvement in economic performance, and also demonstrates the versatility and applicability of the model in various market contexts. Understanding these relationships is crucial for investors and policymakers to make informed decisions and navigate the interdependencies between traditional finance markets and the digital economy.

1 Introduction

DSGE econometric modeling applies general equilibrium theory and microeconomic principles in a tractable manner to advance economic phenomena, such as economic growth and business cycles, as well as policy effects and market shocks.

A major perspective in explaining shocks' impact and influence on capital flows across markets is to recognize that markets always tend for equilibrium even though frictions can persist, influence, and derail existing projections and directions.

The outcome of any important macroeconomic policy change is the net effect of forces operating on different parts of the economy. A central challenge facing policymakers is how to assess the relative strength of those forces. Dynamic stochastic general equilibrium models are the leading tool for making such assessments in an open and transparent manner (Christiano et al., 2018).

Most of the researchers use the DSGE term in business cycle fluctuations or quantitative models of growth. The Real Business Cycle (RBC) model is an eloquent example associated with Kydland Finn E. & Edward C. Prescott (1982) and Long J. B. & Plosser C.I. (1983). The basis of the early RBC models was constructed on the assumption that the economy was populated by households that took part in perfect competitive goods, labor, and asset markets. These models assumed that aggregated fluctuations in the economy are an efficient response to the economy to the source of uncertainty, along with the exogenous technological shocks. Christiano et al. (2018)

expressed their opinion on the inefficiency of RBC models from three variables: 1. Micro data, which can influence some of the factors involved in the model, such as credit, insurance, and labor market; 2. The equity premium, which was not properly assessed, as its volatility was not taken into account; 3. The money factor, which was not integrated into the model, thus it provided inconsistencies with the various historical events, such as the US recession from the 1980s, which was predominately provoked by monetary factors.

New Keynesian DSGE models have been adapted from the traditional RBC models, taking into account some of their nominal frictions, in terms of labor and goods markets. Kolosa et al. (2012) study stated that the DSGE model proposed by Christiano et al. (2005), which is based on the theory of Smets and Wouters (2003), using Bayesian techniques, is considered to be a benchmark DSGE model for a closed economy.

The concept behind this model is also based on the thesis of Friedmanite M. (1968), as it assumes that in the long run, the monetary policy does not affect real variables such as output and interest rate. The fundamental view of this theory has its foundation in the fact that a transitory fall in nominal interest rate (policy-induced) is associated with a decline in real interest rate, an expansion of economic activity, and a small to moderate rise in inflation.

This study of financial digital assets and their interactions with traditional financial markets holds significant importance in today's rapidly evolving financial landscape. A DSGE analysis provides a valuable framework to comprehend the complexities and interdependencies between digital assets and the broader financial system. By incorporating digital assets into the DSGE model, researchers and policymakers gain insights into how these assets affect macroeconomic variables, financial stability, and policy outcomes. Understanding the implications of digital assets on traditional financial markets is crucial for policymakers to develop appropriate regulatory frameworks, market participants to make informed investment decisions, and central banks to manage monetary policy effectively. A DSGE analysis enables the examination of the transmission mechanisms through which shocks originating from digital assets propagate through the economy, shedding light on potential risks, spillover effects, and the overall resilience of the financial system.

This article is structured into several sections, each contributing to a comprehensive analysis of the relationship between financial digital assets and traditional financial markets using an adapted New Keynesian model. The study provides an overview of DSGE models and their significance in macroeconomic analysis, as it explores variations of DSGE models and their applications to financial digital assets. The materials and methods section presents the adaptation of the New Keynesian model for this study, including data sources, variables, and estimation techniques. Furthermore, by integrating VAR modeling into the DSGE framework, the analysis captures the dynamic nature of the relationships between the markets, leading to more reliable and interpretable results that provide a comprehensive understanding of their interconnected dynamics. The results and discussions section presents empirical findings, analyzing the relationships between the markets, and examining the impact of shocks and economic variables on market behavior. The conclusion summarizes the key insights derived from the analysis, discussing their implications and identifying potential avenues for future research.

Consequently, such analysis contributes to the advancement of knowledge in the field and aids in developing robust strategies for navigating the evolving landscape of financial digital assets.

2 Literature Review

New variations of the New Keynesian DSGE (NK-DSGE) were proposed by Yun T. (1996), Clarida et al. (1999), and Woodford (2003), which explored the Fisherian (Fisher) and anti-Fisherian properties. This property satisfies that permanent changes in the monetary policy induce important changes in the inflation and nominal interest rate; and anti-Fisherian property focuses on the transitory changes in monetary policy which can produce fluctuations in the nominal interest rate, along with the inflation rating, but of the opposite sign.

The DSGE models have been subjected to some negative opinions by New-Keynesian, also known as Neo-Keynesian, economists like Blanchard (2018) and Stiglitz (2018). Blanchard's study (2018) has revealed that the aggregated demand is derived as a consumption demand for infinitely long-lived and foresight consumers, as its implications for the degree of foresight and the role of the interest rate are important. He continues to argue that price adjustment is characterized by forward-looking inflation, but in these models, this factor is overlooked, as it is not captured in the fundamental inertia if inflation occurs. The equation based on the behavior of consumers is known as the "Euler equation" and the equation which is focused on the behavior of prices is derived from the research of Calvo G.A. (1983) and is known as the "Calvo pricing". On the same note, Stiglitz (2018) comments that there is no scientific basis for one particular set of movements over another. His research has as a foundation the pre-crisis DSGE models which did not allow for financial frictions and liquidity-constrained consumers. Gali et al. (2007) debated this analysis in their research as their findings present that liquidity constraints magnify the effects of government spending.

Asimakopoulos et. al (2019) applied a Dynamic Stochastic General Equilibrium model to evaluate the economic repercussions of crypto assets. They assumed that crypto assets offer an alternative currency option to government currency, with an endogenous supply and demand. The results indicate a substitution effect between the real balances of government currency and crypto in response to technology, preferences, and monetary policy shocks. In addition, real balances of crypto assets, exhibit a countercyclical reaction to these shocks. Moreover, they analyze concluded that government currency demand shocks have larger effects on the economy than shocks to crypto assets demand. Their results also show that crypto assets productivity shocks have negative effects on output and on the exchange rate between government currency and cryptocurrencies, with a more pronounced negative reaction to output if the central bank increases its weight to government growth. Overall, their research provides novel insights into the underlying mechanisms of crypto assets and spillover effects on the economy.

The role of government-issued currency in the economy has been a topic of study in many Dynamic Stochastic General Equilibrium (DSGE) models. One example is the research conducted by Nelson in 2002, which provided empirical data for the United States and the United Kingdom to support the idea that the growth of the real money base has a significant impact on real economic activity. Specifically, Nelson's findings indicate that when prices are rigid, including a long-term nominal interest rate in the money demand function amplifies the effect of changes in the nominal money stock on real aggregate demand.

The research performed by Clemens Sialm (2006) analyzed the effects of stochastic taxation on asset prices in a dynamic general equilibrium model. The paper generalizes the Lucas (1978) asset pricing model by introducing a flat consumption tax, which follows a two-state Markov chain. This tax does not merely affect equity securities: it affects all assets symmetrically. Whenever taxes change, asset prices need to adjust instantaneously to clear asset markets. These price changes increase the variability of expected and actual asset returns. These price adjustments affect assets with long durations, such as equities and long-term bonds, more than short-term assets. As a conclusion of the research performed by Clemens (2006), under favorable conditions, investors require higher term and equity premia as compensation for the risk introduced by tax changes.

This model has been applied to other sectors and markets like Yang et al. (2020), who researched a DSGE model with regard to the COVID-19 pandemic's impact on the tourism sector. Their model is generalizable to any epidemic and it supports the policy of providing tourism consumption vouchers for residents.

Olatunji A. S. & Oladimeji T. S. (2019) analyzed the impact of energy policy in curbing the effect of carbon emissions in the United States, China, and Nigeria. Their DSGE study provides a great insight into carbon emission disclosure within the context of uncertain productivity and it interacts with the components of energy production function with optimal policy rules. The findings suggest production uncertainty exists in the present energy mix and the present carbon tax levied on emitters is not a true disclosure of current pollution. They conclude that the monetary stance can act as a catalyst for minimizing carbon through incentives to invest in renewable energy and that policy direction towards a carbon-free environment, when properly channeled, would impact positively on decarbonization.

Liu A et al. (2018) performed research on a two-sector, small, open economy, which is modeled under the dynamic stochastic general equilibrium framework. The model is estimated using the Bayesian method based on real tourism and macroeconomic data from Mauritius for the period from 1999 to 2014. The impulse response functions are used to simulate the contribution of tourism to economic growth when there is a productivity shock in the tourism sector. The simulation results show that the Mauritian GDP would increase by 0.09% if the productivity of tourism is improved by 1%, indicating that tourism could lead to economic growth.

3 Materials and Methods

For the purposes of our research, we will utilize the methodology of the New Keynesian model as a foundation. DSGE models that are classified as "New Keynesian" are a subclass of DSGE models that are based on the assumption that prices and wages are "sticky," meaning that they are slow to adjust to changes in the economy. This assumption is based on the idea that firms and workers may be unwilling or unable to adjust their prices or wages in the short run, leading to persistent deviations from equilibrium. New Keynesian DSGE models are often used to study how central banks, such as the Federal Reserve, can influence the economy through the use of monetary policy tools, such as interest rates. Overall, New Keynesian DSGE models serve as a valuable instrument for comprehending and examining the dynamics of economies in the short term. However, it is crucial to acknowledge that these models rely on assumptions that may not universally apply to real-world conditions. Therefore, when interpreting the outcomes of these models, it is essential to consider them in conjunction with additional economic evidence and analysis, ensuring a comprehensive understanding of the complex dynamics at play.

In our economy, households play a significant role as they shape the economy through their actions such as consumption, labor, savings, and investments.

Households can be thought of as infinite-lived agents, like families, which evolve on some basic elementary actions like the ones to consume, rest and work. The utility factor they derive comes from consumption and leisure. They offer labor to producing firms in exchange for money, which they can then use to consume. We can also consider that households can invest their savings into capital without incurring any costs, and in return, firms receive returns on their investments.

The household's optimization problem can be written as follows:

$$\max_{C_t,L_t} E_t \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \quad \beta > 0$$

(1) – Maximization of Households problem

Where: C_t – Consumption; E_t – Expected utility over time; β^t – Intertemporal discount factor; u – Utility Function.

For this study, we will utilize the log-log utility function to model the utility of households. The log-log utility function has been widely used in economic modeling as it exhibits properties such as constant elasticity of substitution, which allows for tractable analysis of optimization problems. By maximizing this utility function subject to the household's budget constraint, we can derive the optimal consumption and leisure choices for the household, providing insight into the household's demand for goods and services in the economy:

$$\max_{C_t,L_t} E_t \sum_{t=0}^{\infty} \beta^t [\log(C_t) + \gamma \log(L_t)], \quad \beta > 0$$

(2) – Maximization of Household problem with utility function

Where: L_t – Leisure; γ – Leisure share parameter.

The leisure share parameter represents the proportion of time that households allocate to leisure activities rather than consuming or working.

Households strive to maximize their utility by consuming as much as possible, but this is subject to certain constraints. For example, if a household does not have sufficient capital, it may not be able to complete a transaction. Similarly, if a household works all the time, they may not have sufficient time for leisure activities. These limits will be considered in the model:

$$C_t + S_t = W_t H_t + R_t K_t$$

(3) – Constraints for Households problem

Where: C_t – Consumption; S_t – Savings; W_t – Wages; H_t – Hours worked; R_t – Returns on capital; K_t – Capital.

This constraint is a balance of different elements. The left side of the equation represents the household's consumption and savings, which indicate how much they can spend. The right side includes the factors that allow a household to acquire capital, such as working more hours or earning a higher return on capital. It is important to keep in mind that a household can never spend more than they earn.

In this analysis, we have employed the concepts of leisure, savings, and capital. To express these terms mathematically, we can define leisure as the number of hours that a household is not engaged in work:

$$\mathrm{L_t} = 1 - \mathrm{H_t}.$$

(4) – Definition of leisure

We also assume that investments can be made without incurring any costs, so at any given time t, we can consider savings and investments to be equal.

Capital is also defined as the amount of physical assets that a household or firm owns:

$$S_t = I_t$$

(5) – Savings and investment

The evolution of capital can be defined by the following equation:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

(6) – Evolution of capital

This equation states that the amount of capital at time $t + 1$ will depend on the investment made at time t, as well as the capital at time t multiplied by the capital depreciation rate (δ). In other words, the future level of capital is determined by the current level of investment and the rate at which capital depreciates over time.

Replacing equations 5 and 6 in the constraint equation, we got the next equation:

$$C_t + K_{t+1} = W_t H_t + (1 + R_t - \delta)K_t$$

(7) – Restriction equation for the model

This is the new constraint equation we will use to compute our model.

Then we define the maximization problem for households:

$$\max_{C_t L_t} E_t \sum \beta^t \left[log(C_t) \ + \gamma log(1 - H_t)\right]$$

(8) – Defined Household problem

We determine the general formula for Lagrange multipliers:

$$L(x, y, \lambda) = f(x, y) + \lambda g(x, y)$$

(9) – General Expression of Lagrange Multipliers

By substituting these definitions into the model, we obtain the following optimization problem:

$$\max_{C_t, H_t, K_{t+1}} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \{[\log(C_t) + \gamma \log(1 - H_t)] - \lambda_t [C_t + K_{t+1} - W_t H_t - (1 + R_t - \delta)K_t]\}$$

(10) – Lagrange multipliers optimization problem

The challenge we must address is to find the equations that will allow us to analyze our model. To do this, we will use the Lagrange multipliers method, which involves taking the partial derivative of the Lagrange function with respect to each variable in our model. The first partial derivative is:

$$\frac{\partial L}{\partial c_t} = \beta^t \left[\frac{1}{c^t} - \lambda_t\right] = \ 0 \rightarrow \lambda_t = \frac{1}{c_t}$$

(8) – Consumption (First Order Equation)

$$\frac{\partial L}{\partial H_t} = \beta^t \left[-\gamma \frac{1}{(1-H_t)} + W_t \lambda_t\right] = 0 \rightarrow \lambda_t W_t = \frac{\gamma}{(1-H_t)}$$

(9) – Hours worked (First Order Equation)

$$\frac{\partial L}{\partial K_{t+1}} = \beta^t - \lambda_t + \beta^{t+1}\lambda_{t+1}(1 + R_{t+1} - \delta) = 0 \rightarrow \beta^t \lambda^t = \beta^{t+1}\lambda_{t+1}(1 + R_{t+1} - \delta)$$

(10) – Capital Evolution (First Order Equation)

We have derived three equations that must be solved concurrently. By utilizing equations 8 and 9, we can determine the labor supply:

$$\frac{1}{C_t} \mathrm{W}_t = \frac{\gamma}{1 - H_t} \lambda_{t+1}(1 + R_{t+1} - \delta) \rightarrow 1 - H_t = \frac{C_t \gamma}{\mathrm{W}_t}$$

(11) – Replacing equations to solve hours of work

Our next step is determining the solution of H_t:

$$H_t = \frac{W_t - C_t \gamma}{\mathrm{W}_t}$$

(12) – Hours of work equation

By utilizing equations 8 and 10, we can derive the solution for the Euler Equation:

$$\beta^t \frac{1}{C_t} = \beta^{t+1} \frac{1}{C_{t+1}} (1 + R_{t+1} - \delta)$$

(13) – Replacing equation 8 in 10

$$\frac{C_{t+1}}{C_t} = \frac{\beta^{t+1}}{\beta^t} (1 + R_{t+1} - \delta)$$

(14) – Solving for consumption

And finally:

$$\frac{C_{t+1}}{C_t} = \beta(1 + R_{t+1} - \delta)$$

(15) – Consumption over time

We have successfully resolved the optimization problem and obtained the necessary equations. However, an important aspect remains to be addressed, which is the issue of firms' maximization.

Firms' production function is defined as a Cobb-Douglas function:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

(16) – Firms' production function

Where: Y_t – Output; K_t – Capital; H_t – Hours of work; α – Input Share.

In this context, A_t is the stochastic productivity shock which is defined as a first-order autoregressive process:

$$\ln(A_{t+1}) = \rho \ln(A_t) + e_{t+1}, \quad e_{t+1} \sim N(0, \sigma^2)$$

(17) – Autoregressive process for productivity shock

Firms require both labor and capital, but they must cover the costs of obtaining them. Therefore, they are subject to the following restriction:

$$W_tH_t + R_tK_t = 0$$

(18) – Constraint for firms' production problem

Having previously established the definitions of each variable, we can now proceed to formulate a maximization problem for firms. The problem can be stated as follows:

$$\max \pi = A_tK_t^{\alpha}H_t^{1-\alpha}$$

(19) – Maximization problem for firms' profit

The maximation problem will be subject to:

$$W_tH_t + R_tK_t = 0$$

(20) – Constraint for firms' profit problem

Where: π – Firms Profit

The next step in our analysis is to define the final problem which is as follows:

$$\max \pi = A_tK_t^{\alpha}H_t^{1-\alpha} - W_tH_t - R_tK_t$$

(21) – Maximization problem with constraint for firms' profit

To address this issue, we will take the partial derivative of the variables in this problem, which are capital and hours of work. We will begin by taking the partial derivative of capital:

$$\frac{\partial \pi}{\partial K_t} = \alpha A_tK_t^{\alpha-1}H_t^{1-\alpha} - R_t = 0$$

(22) – First order condition Capital

The resolution for R_t will be:

$$R_t = \alpha A_tK_t^{\alpha-1}H_t^{1-\alpha} = \alpha A_t\frac{K_t^{\alpha}H_t^{1-\alpha}}{K_t}$$

(23) – Solving equation 18 for returns

We will continue with the partial derivative of hours worked:

$$\frac{\partial \pi}{\partial H_t} = (1-\alpha)A_tK_t^{\alpha}H_t^{1-\alpha-1} - W_t = 0$$

(24) – First order condition hours of work

Where W_t can be solved as follows:

$$W_t = (1-\alpha)A_tK_t^{\alpha}H_t^{1-\alpha-1} = (1-\alpha)A_tK_t^{\alpha}H_t^{1-\alpha} \cdot H_t^{-1} = \ \dots$$

$$\dots = \frac{(1-\alpha)A_tK_t^{\alpha}H_t^{1-\alpha}}{H_t}$$

(25) – Solving equation 20 for Wages

At this stage, we note that the enumerator of equation 16 is the production function, so we can rewrite R_t as:

$$R_t = \alpha\frac{Y_t}{K_t}$$

(26) – Replacing firms' production function on returns

It is important to note that equation 16 includes the production function in the numerator, therefore we can express wages as:

$$W_t = (1-\alpha)\frac{Y_t}{H_t}$$

(27) – Replacing firms' production function on wages

With this, we have completed the Dynamic Stochastic General Equilibrium model, and have obtained the necessary equations for solving it. Now, we must define a steady-state, which entails identifying a combination of *C*, *L*, *K*, *H*, and a set of prices *R* and *W* that fulfill the following criteria.

Three hypotheses have been proposed to explain the behavior of households and firms in the economy:

1. Households seek to maximize their utility by carefully choosing the optimal levels of consumption and leisure, based on their available income and the rate of return on their savings;
2. Firms aim to maximize their profits by selecting the optimal levels of labor and capital, considering the current wage rate and the cost of capital;
3. The use of resources in the economy is efficient and effective, meaning that they are utilized in a way that maximizes their potential value.

If these hypotheses hold, then the dynamic equations must be written as follows:

$$Y_t = C_t + I_t$$

(28) – Output of the model

Equation 28 represents the output of an economy, which is the total value of goods and services produced in each period *t*. Output is equal to the sum of capital *C*, the physical goods and assets used in production, and investment *I*, the purchase or expansion of capital goods. Capital and investment play a crucial role in determining the output, as they can increase an economy's capacity to produce goods and services, potentially leading to economic growth.

$$K_{t+1} = I_t + (1-\delta)K_t$$

(29) – Evolution of capital

The 29th equation represents the evolution of capital over time. It shows how capital changes from one period t to the next $t+1$, based on the level of investment I_t and the depreciation of existing capital δ.

$$\frac{C_{t+1}}{C_t} = \beta[R_{t+1} + 1 - \delta]$$

(30) – Consumption over time

Equation 30 illustrates the consumption behavior of households. It illustrates the correlation between current consumption and future consumption, taking into account anticipated returns on wealth, the preference for consuming now rather than later, and the depreciation of assets.

$$H_t = \frac{W_t - \gamma C_t}{W_t}$$

(31) – Hours of work

Equation 31 represents the number of hours an individual will choose to work in a given period. It is influenced by wages, leisure preferences, and consumption needs. By analyzing this equation, we can gain a better understanding of how individuals make choices regarding work and leisure.

$$R_t = \alpha\frac{Y_t}{K_t}$$

(32) – Returns over time

Equation 32 represents the returns over time *R*, of a particular investment or asset. These returns are influenced by the input share α, as well as the output *Y* and capital *K* of the investment or asset in question. By examining this equation, we can gain insights into the factors that determine the returns of an investment over time and how these returns may vary depending on the input share, output, and capital involved.

$$W_t = (1 - \alpha)\frac{Y_t}{H_t}$$

(33) – Wages over time

Equation 33 shows how wages W_t are affected by the input share α, output Y_t, and labor hours H_t of an individual. This equation helps us understand the factors that influence an individual's wages and how these wages may change based on the input share, output, and labor hours.

$$Y_t = A_t K_t^{\alpha} H_t^{(1-\alpha)}$$

(34) – Firms' production function

Equation 34 represents a production function, which shows the relationship between inputs (capital and labor) and output. The technological factor A_t, captures the efficiency with which the inputs are used to produce output, and the input share, alpha (α), reflects the relative importance of capital and labor in the production process. If we consider that *A* is a productivity shock, it means that *A* represents a sudden change in the efficiency of production. Productivity shocks can be positive (increasing the efficiency of production) or negative (decreasing the efficiency of production) and can be caused by a variety of factors such as technological innovations, changes in the availability or cost of inputs, or shifts in consumer demand. Examining the impact of productivity shocks on the production function can help us understand how these shocks may affect output and the allocation of resources in the economy.

$$\ln(A_{t+1}) = \rho \ln(A_t) + e_{t+1}$$

(35) – Autoregressive process of the productivity shock

Equation 35 represents an autoregressive model of productivity shock, which can be used to forecast the future level of productivity based on past levels and the persistence of the shock. In this equation, A_t represents the level of productivity at time *t*, and A_{t+1} represents the level of productivity at time $t + 1$. The parameter rho (ρ) captures the persistence of the shock or the degree to which the current level of productivity is influenced by past levels. The error term, e_{t+1}, represents any unpredictable or stochastic factors that may affect the level of productivity. This model can be useful for understanding how the persistence of productivity shocks may affect the level of output in the economy.

Autoregressive Processes and Shocks

An Autoregressive process is a statistical model used to explain temporal dependencies in a time series, and it assumes that the current value of the time series can be predicted based on its past observations. It is a stationary time series that can be represented by the following equation:

$$y(t) = c + a_1 y(t-1) + a_2 y(t-2) + \cdots + a_p * y(t-p) + e(t)$$

(36) – General equation for autoregressive models

Where *y(t)* is the current value of the time series, *c* is a constant term, a_p are coefficients and *e(t)* is a white noise error term, this means that the distribution of errors is normal.

Within Autoregressive modeling, *p* defines the order of the process, as this determines the number of lagged periods of the time series used to make the prediction. If *p* has a higher value, the model is more complex capturing more dependencies in data, but this also develops a trend that can overfit.

A shock is an exogenous event that affects the economy and causes a deviation from the equilibrium path. Shocks could be positive or negative, depending on whether they cause the economy to move closer to equilibrium or not.

An exogenous event is a change or occurrence that takes place outside of a system and influences that system. Exogenous events are often unpredictable and may be referred to as “shocks” because they can disrupt the functioning of the system.

Shocks are often incorporated into economic models to analyze the effects of these external influences on the economy. Economic models such as DSGE models can be used to examine the response of the economy to these shocks and to evaluate the effectiveness of different policy responses to them.

Many different types of shocks can be included in a DSGE model, including:

- Monetary shocks: Changes in monetary policy, such as changes in interest rates or the money supply, that affect the economy's financial conditions;
- Fiscal shocks: Changes in government spending or taxation that affect the level of aggregate demand in the economy;
- Technology or Productivity shocks: Changes in the level of productivity or technological progress that affect the economy's potential output;
- Trade shocks: Changes in the terms of trade, such as changes in the prices of exports or imports, that affect the economy's balance of payments;
- Supply shocks: Changes in the availability of resources, such as oil or natural disasters, that affect the economy's production capabilities.

Shocks and Autoregressive (AR) processes are often studied together in the context of time series analysis. A shock can be thought of as a sudden, transitory deviation from the trend or long-run behavior of a time series. An AR process, on the other hand, is a statistical model that describes the temporal dependencies in a time series by assuming that the current value of the series can be predicted based on its past values.

In the context of an AR process, a shock can be represented by an exogenous error term in the model. For example, consider the following AR (1) model:

$$y(t) = c + ay(t-1) + e(t)$$

(37) – Autoregressive model first order

By analyzing the impact of shocks on an AR process, it is possible to study how the time series responds to these deviations from its trend and to evaluate the persistence of the shocks over time. This can be useful for a wide range of applications, such as forecasting future values of the time series or evaluating the effectiveness of policy interventions.

In DSGE models, exogenous shocks are incorporated using an autoregressive process with an error term that represents the stochastic component of the model.

Calibration and settings of variables

We will calibrate our model using parameters that have been widely used in the academic literature. We are using this approach because we are dealing with many markets and want to achieve a general and consistent response in our results.

Table 1 - Model Calibration

Variable	β	α	δ	γ
Value	0.98	0.33	0.028	3.3

Table 2 – Variables definition

Variable Type	Unobserved	Observed	Endostate	Exostate

Variable Name	- Consumption (*C*) - Model Output (*Y*) - Work Hours (*W*) - Investment (*I*)	- Returns (*R*)	- Capital (*K*)	-Productivity Shock (*A*)

Steady-state

In the context of Dynamic Stochastic General Equilibrium modeling, the steady-state refers to a long-run equilibrium condition in which the economy's key macroeconomic variables, such as output, employment, and inflation, remain constant over time. In other words, the steady-state represents a hypothetical scenario in which the economy has reached a point of balance and is not experiencing any growth or decline. This equilibrium is often used as a benchmark for policy analysis and to evaluate the long-run effects of changes in economic policy. Thus, the steady-state plays a central role in DSGE modeling, serving as a reference point for evaluating actual economic performance and assessing the potential implications of various policy options.

Impulse Response Functions and Forecast

The Impulse Response Function (IRF) of a dynamic system is a function that describes the output of the system for a brief "impulse" input. Mathematically, it is defined as the response of a system to a delta function input, which is a function with an infinitely high amplitude at zero time, and zero amplitude everywhere else.

$$h(t) = L^{-1}H(s)$$

(38) – Impulse Response function

Within equation 38, *h(t)* is the impulse response function, L^{-1} is the inverse Laplace transform and *H(s)* is the transfer function of the system.

Another way to express the impulse response function is:

$$h(t) = \int_{-\infty}^{\infty} h(\tau)\delta(t-\tau)d\,\tau$$

(39) – Impulse Response function

In this context, *h(t)* is the output, *h(τ)* is the impulse response of the system, $\delta(t-\tau)$ is the Dirac delta function, and the integral is considered over all time.

It can be intuitively interpreted as the response of the system when the input is infinitely short and an infinitely large amplitude (or equivalently a unit impulse) input is given at *t = 0*.

In econometrics, we use IRF to describe the effect of a shock or disturbance on a system over time. It shows how a variable of interest responds to a one-time shock to another variable in the system.

For example, if a central bank increases the interest rate, this could be viewed as a shock to the economy. An impulse response function could be used to understand how this shock to the interest rate affects other variables in the economy, such as inflation or economic growth, over a given period of time.

In addition to being used to understand the effects of a shock on a system, impulse response functions can also be used to forecast the future response of a system to a shock, or to evaluate the effectiveness of policy interventions.

Vector Autoregression Model - VAR

Vector Autoregressive models are a generalization of univariate autoregressive models that allow for the analysis of multivariate time series of any order. A univariate regression is a one-equation model in which the current value of a single variable is explained by its lagged values. In contrast, a VAR model is a system of *n*-variables and *n*-equations that express each variable as a linear function of its own past, the past values of all other variables, and a serially uncorrelated error term. Therefore, VAR models offer a more comprehensive approach to the analysis of multivariate time series data.

VAR models are widely used in forecasting and analyzing the interactions between variables over time. These models provide insight into the underlying causes of a time series, allowing us to interpret the approximations as effects of the model. In addition, they can be used to identify the relationship between different variables and to analyze the impact of exogenous factors on the system. Overall, VAR models are a powerful tool for understanding and predicting the behavior of complex systems.

A bivariate VAR(1) model is defined as a system of two variables, each of which is a function of its own past and the past of the other variable. The notation VAR(1) indicates that the model is based on the first order of lag, meaning that the current value of each variable is explained by the value of the same variable in the previous period, and the value of the other variable in the same period.

Additionally, as we increase the complexity of the model, we can add more variables and lags, which will give us more insight into the relationships between variables and the dynamics of the system.

We define a bivariate VAR(1) as follows:

$$y_t = a_1 + b_{11}y_{t-1} + b_{12}x_{t-1} + u_t$$

(40) – VAR for the y variable

$$x_t = a_2 + b_{21}y_{t-1} + b_{22}x_{t-1} + v_t$$

(41) – VAR for the x variable

We can also write the matrix representation as follows:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \end{bmatrix}$$

(42) – Matrix representation of VAR for x and y

In the provided Vector Autoregressive (VAR) model, there are various parameters and variables that play essential roles in capturing the linear relationships between multiple time series. The model consists of two endogenous variables, y_t and x_t, which are the main focus of the analysis. The coefficients a_1 and a_2 represent constant terms in the model, capturing the baseline levels of y_t and x_t, respectively.

The parameters in matrix B represent the linear relationships between the lagged values of the endogenous variables. Specifically, b_{11} and b_{22} denote the autoregressive coefficients for y_t and x_t, capturing the direct impact of their own lagged values. Meanwhile, b_{12} and b_{21} represent the cross-variable coefficients, illustrating how the lagged value of one variable influences the other.

Additionally, the model incorporates two exogenous variables, u_t and v_t, which are typically assumed to be random shocks or error terms. The exogenous variables account for unexplained variations in y_t and x_t, allowing the model to better represent the inherent complexities and uncertainties in real-world data.

Using matrix representation makes it easier to generate and solve larger VAR models computationally. By expressing the model in a matrix form, it is possible to extend the model to include more variables and lags, without having to write out a separate equation for each variable. This not only simplifies the notation but also allows for more efficient computation of the model's parameters. It's important to note that as the complexity of the model increases, the computational resources required to solve the model also increase and it's important to choose the right method to estimate the model accordingly.

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_k \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ c_k \end{bmatrix} + \begin{bmatrix} \phi_{11} & \cdot & \cdot \\ \phi_{21} & \cdot & \cdot \\ \cdot & & \\ \cdot & & \\ \phi_{k1} & \cdot & \cdot \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \cdot \\ \cdot \\ y_{k,t-1} \end{bmatrix} + \cdots . + \begin{bmatrix} \phi_{11} & \cdot & \cdot \\ \phi_{21} & \cdot & \cdot \\ \cdot & & \\ \cdot & & \\ \phi_{k1} & \cdot & \cdot \end{bmatrix} \begin{bmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \cdot \\ \cdot \\ y_{k,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \varepsilon_k \end{bmatrix}$$

(43) – Matrix representation for generalized VAR

The given equation represents a generalized Vector Autoregressive (VAR) model of order *p*, which is a statistical model designed to capture the linear interdependencies among multiple time series variables. In this VAR(*p*) model, there are *k* endogenous variables, each influenced by their past values and the past values of the other variables in the system, up to p lags. The model includes a *k*-dimensional vector of constant terms, which represent the baseline levels of the endogenous variables. Additionally, there are $k \times k$ matrices of coefficients for each lag, which capture the linear relationships between the variables at different time lags. Lastly, the model incorporates a *k*-dimensional vector of error terms or random shocks, accounting for unexplained variations in the endogenous variables. This structure allows the VAR(*p*) model to represent the inherent complexities and uncertainties present in real data by incorporating both the historical influences of the variables and external random shocks.

It is crucial to ensure that the variables y_t and x_t, as well as any other variables included in the VAR model, are stationary before estimating the model. This can be done by applying statistical tests such as the Augmented Dickey-Fuller (ADF) test. Additionally, the VAR model includes error terms, represented by u_t and v_t, which are assumed to be white noise disturbances, also known as shock terms. The coefficients in the main matrix of the VAR model are then estimated by the Ordinary Least Squares (OLS) method.

The null hypothesis of the ADF test indicates that there is a unit root in the time series, which means that the time series is non-stationary and has a trend. The alternative hypothesis is that there is no unit root, and the time series is stationary.

Ordinary Least Squares (OLS) is a statistical technique used to estimate the parameters of a linear regression model. In a linear regression model, the relationship between the independent variables and the dependent variable is represented by a linear equation. OLS finds the values of the coefficients (i.e., the parameters) of the equation that minimize the sum of the squared differences between the predicted values of the dependent variable and the actual values of the dependent variable.

OLS assumes that the errors (i.e., the differences between the predicted values and the actual values) are normally distributed with a mean of zero and a constant variance. It also assumes that there is no multicollinearity among the independent variables. Under these assumptions, the OLS estimator is unbiased, efficient, and has the smallest variance among all linear unbiased estimators.

OLS can be used for both simple and multiple linear regression. The process of finding the optimal coefficients is the same in both cases, but the algebraic calculations are more complex when there are multiple independent variables.

In practice, OLS is widely used and is often the default method for fitting linear regression models. The estimates obtained by OLS are also used as starting values for more sophisticated methods such as weighted least squares, and non-linear least squares.

Utilizing price returns in a Vector Autoregression (VAR) model when analyzing multiple time series prices offers several advantages, leading to more reliable and interpretable results. First, price returns help to address the non-stationarity issue common in financial time series data. Raw price data often exhibit trends and seasonal patterns that violate the stationarity assumption in time series analysis. By using price returns, we transform the data into a stationary format, mitigating the risk of spurious correlations and improving the model's overall performance. Second, price returns facilitate comparability between different assets or instruments, as they normalize price movements by representing them as percentage changes. This enables us to compare returns across assets with varying price levels and scales more effectively. Lastly, using price returns aligns with economic theory and market practice, as investors and portfolio managers typically focus on returns rather than absolute price levels when making investment decisions. Thus, using price returns in a VAR model allows for more accurate, robust, and relevant analysis, leading to improved insights into the dynamic relationships between the time series prices under consideration.

In our study, the endogenous variables refer to the relationship of the prices, and the exogenous variables refer to the shocks. This distinction is crucial in our VAR model, as endogenous variables are determined within the system or model and are influenced by other variables in the same system. Meanwhile, exogenous variables are determined outside the system or model and are not influenced by other variables in the system, shocks have a stochastic component.

4 Results and discussions

For our analysis, we have used the historical price data from various financial digital assets and traditional finance markets, with a quarterly time frame from January 2016 to October 2022. The data applied in our research refers to the following assets and markets: Bitcoin (BTC), Ethereum (ETH), Dow Jones Industrial Average (DJIA), Standard and Poor's 500 Index (SP500), Nasdaq Composite (IXIC), Euronext 100 (N100), Nikkei Index (N225), Shanghai Stock Exchange Composite Index (SSE) and Hang Seng Index (HSI). Upon analysis, we found that none of these assets exhibit stationary behavior, meaning that they do not show a consistent mean or constant variance over time.

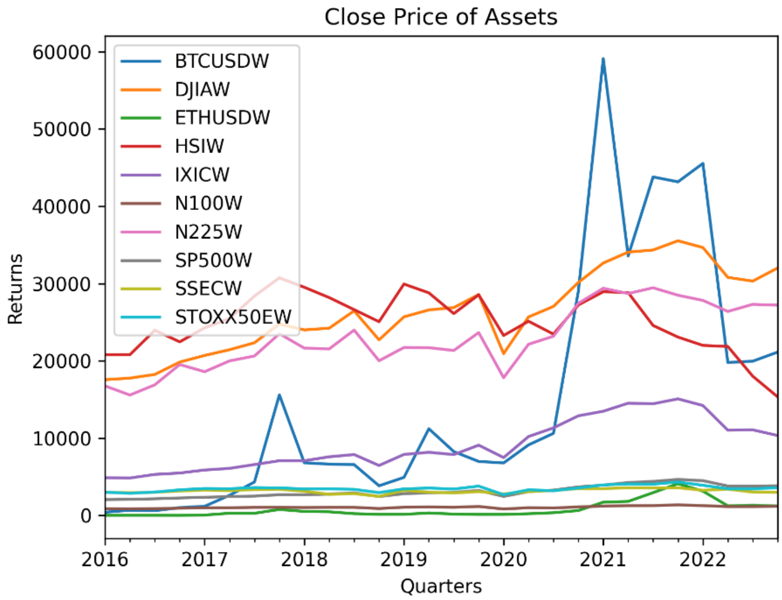


Figure 1 – Close prices of assets

To address this issue, we utilize logarithmic price returns for each time series to identify and account for non-stationarity.

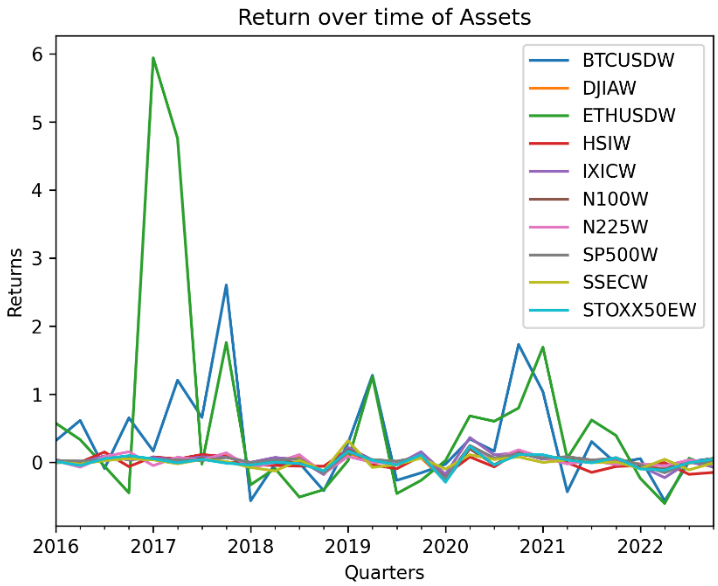


Figure 2 – Price returns of assets

The time series in this analysis demonstrates a clear pattern of mean and constant variance over time. However, it is evident that financial digital assets exhibit higher levels of volatility during certain periods, which is a common characteristic of this type of asset. Despite this, we can still observe the underlying mean and constant variance in the data, so we will use this as input for our model.

Each solution we present will include its estimates and forecasts, as well as a calibration that is consistent across all the solutions. The only elements that will vary from one solution to another are the estimates of the productivity shock and the steady-state, which we will present in tables and illustrate with corresponding graphs.

To accurately analyze this data, it is essential to comprehend the concept of the impulse-response effect. This effect describes the output of a given equation when specific values are applied to the impulse.

In a DSGE model, it is commonly expected that a productivity shock impulse will result in an increase in the magnitude of the other variables. However, as the shock dissipates over time, the effects on the variables will also disappear. It is possible to observe some permanent changes in the variables as a result of the productivity shock impulse.

An increase in productivity within firms leads to an increase in output in a DSGE model. This is due to households optimizing their utilities and subsequently demanding more capital and labor. As a result, salaries also increase. Additionally, the increase in capital leads to an increase in investment and consumption. Furthermore, higher capital leads to higher depreciation rates, requiring a higher investment rate to maintain a constant level of capital. To summarize, a productivity shock results in an overall increase in inputs within the economy.

As we are addressing financial digital assets, we are engaging a digital economy, where we need to find a counterparty of each variable on the model. In this scenario, instead of households, we have miners that try to maximize their utility trough hours of mining and processing power, the maximization of this variable influences increasing salaries and returns as a utility function, and the impulse response for the digital economy should have a similar effect as per the traditional financial economy.

Bitcoin mining is a crucial aspect of the crypto assets ecosystem, as it helps to validate transactions and maintain the integrity of the blockchain. However, for the broader ecosystem to thrive, it is essential for miners to also spend their accumulated crypto assets on goods and services. This aligns with the original vision of bitcoin as a currency that could be used as payment in developing a digital economy parallel to the traditional one. When miners spend their bitcoin, it helps to drive demand for the currency and increases its utility. Additionally, firms that accept bitcoin as a form of payment can also benefit from increased demand and potentially see an increase in profits. As such, the use of the DSGE model to analyze the behavior of bitcoin miners as households and digital economy is a legitimate approach, as it considers the economic principles that drive the adoption and use of the currency in an economy. The same interpretation is valid for Ethereum with its asset Ether (ETH), in the same context.

For each firm, we have an estimation of productivity shock, represented as *A*, so we perform a z-test to measure the significance of the estimation, in order to define the null and alternative hypothesis, and within each case, the conclusion of the statistical test is reported.

$$H_0: A = 0$$

(44) – Null Hypothesis of Productivity Shock Test

$$H_a: A \neq 0$$

(45) – Alternative Hypothesis of Productivity Shock Test

The decision condition consists of the value of the p-value of tests. If the p-value is lower than 0.05, then we reject the null hypothesis, and on the other hand, if they are greater, we cannot reject the null hypothesis, meaning in the last case that our variable is not significant.

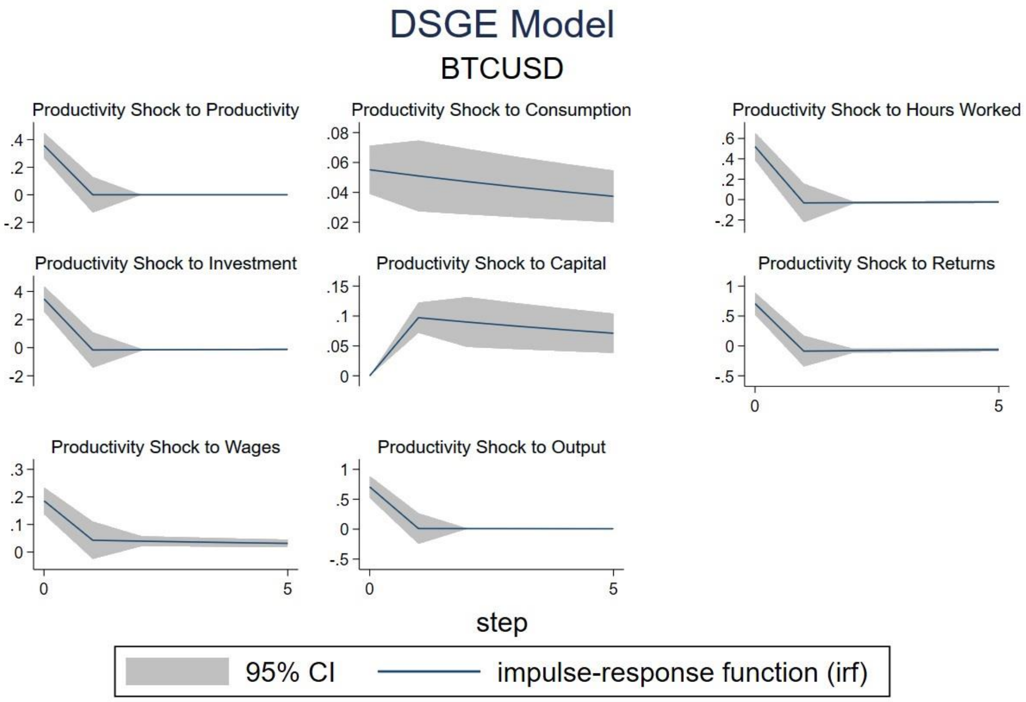


Figure 3 – DSGE Model - BTCUSD

Table 3 – Impulse Response Table BTC

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.337	0.666	3.266	0.052	0.49	0.666	0.175	0
1	0.006	0.021	-0.096	0.049	-0.022	-0.07	0.043	0.091
2	0	0.009	-0.143	0.045	-0.029	-0.077	0.038	0.086
3	0	0.008	-0.133	0.042	-0.027	-0.071	0.035	0.08
4	0	0.008	-0.123	0.039	-0.025	-0.066	0.033	0.074
5	0	0.007	-0.114	0.036	-0.023	-0.061	0.03	0.068

Table 4 – Steady-state BTC

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval

Productivity Shock	0.17	0.168	0.09	0.926	(-0.348, 0.383)

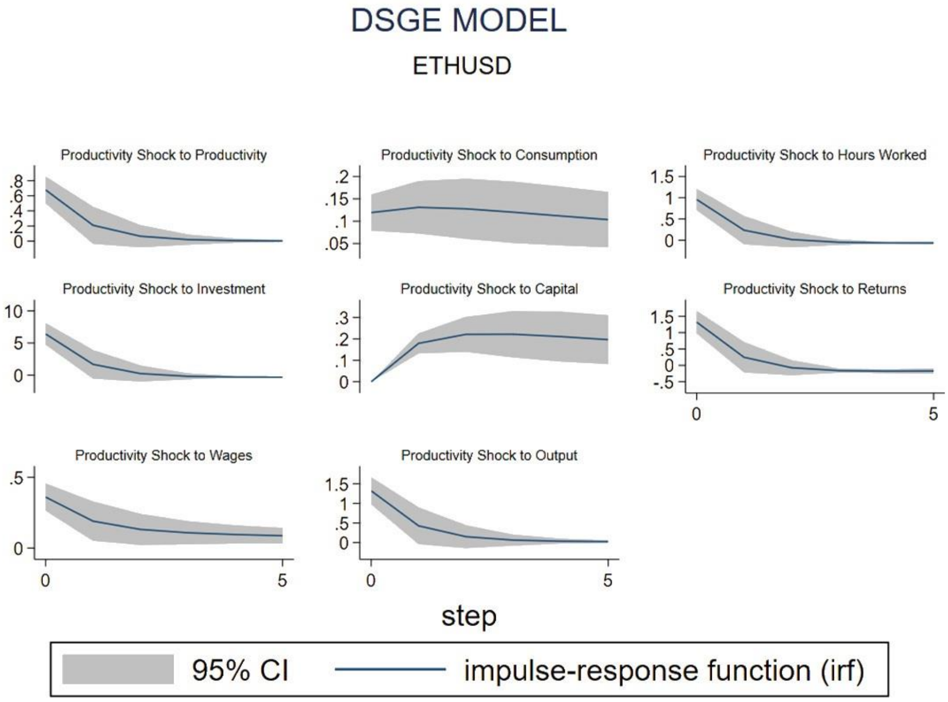


Figure 4 – DSGE Model - ETHUSD

Table 5 – Impulse Response Table ETH

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.677	1.32	6.407	0.119	0.96	1.32	0.36	0
1	0.21	0.427	1.684	0.131	0.237	0.248	0.191	0.179
2	0.065	0.15	0.244	0.128	0.018	-0.072	0.132	0.222
3	0.02	0.063	-0.181	0.12	-0.046	-0.16	0.109	0.222
4	0.006	0.034	-0.294	0.112	-0.062	-0.177	0.096	0.211
5	0.002	0.024	-0.311	0.104	-0.063	-0.172	0.088	0.197

Table 6 – Steady-state ETH

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	0.3	0.177	1.75	0.081	(-0.038, 0.66)

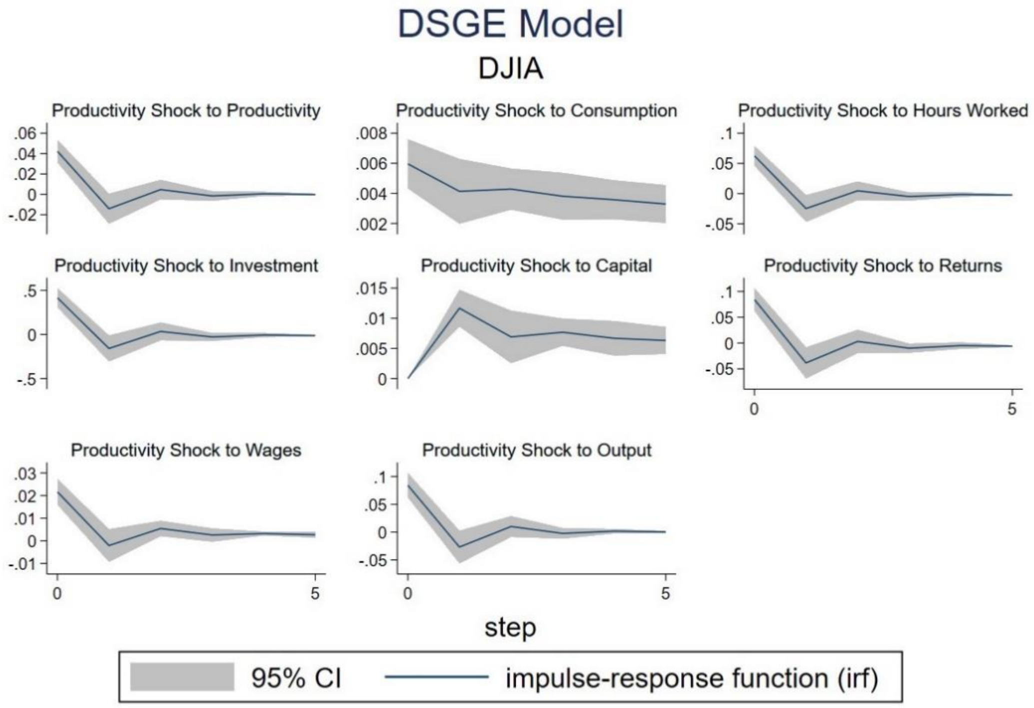


Figure 5 – DSGE Model - DJIA

Table 7 – Impulse Response Table DJIA

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.04	0.079	0.391	0.006	0.059	0.079	0.02	0
1	-0.013	-0.024	-0.145	0.004	-0.023	-0.035	-0.002	0.011
2	0.004	0.009	0.03	0.004	0.004	0.002	0.005	0.007
3	-0.001	-0.002	-0.025	0.004	-0.004	-0.009	0.002	0.007
4	0	0.002	-0.006	0.003	-0.001	-0.005	0.003	0.006

5	0	0	-0.011	0.003	-0.002	-0.006	0.003	0.006

Table 8 – Steady-state DJIA

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.32	0.174	-1.85	0.064	(-0.66, 0.18)

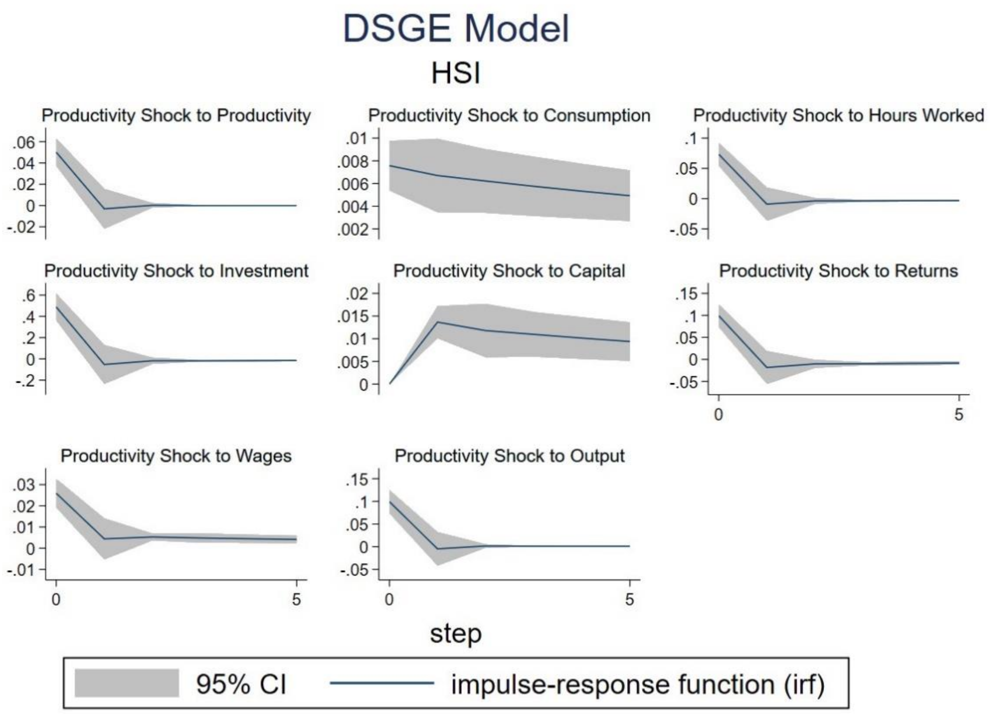


Figure 6 – DSGE Model – Hang Seng Index

Table 9 – Impulse Response Table HSI

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital

step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.048	0.095	0.464	0.007	0.07	0.095	0.025	0
1	-0.001	-0.001	-0.034	0.007	-0.006	-0.014	0.005	0.013
2	0	0.001	-0.019	0.006	-0.004	-0.01	0.005	0.012
3	0	0.001	-0.018	0.006	-0.004	-0.01	0.005	0.011
4	0	0.001	-0.017	0.005	-0.003	-0.009	0.004	0.01
5	0	0.001	-0.015	0.005	-0.003	-0.008	0.004	0.009

Table 10 – Steady-state HSI

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.0269	0.193	-0.14	0.88	(-0.406, 0.35)

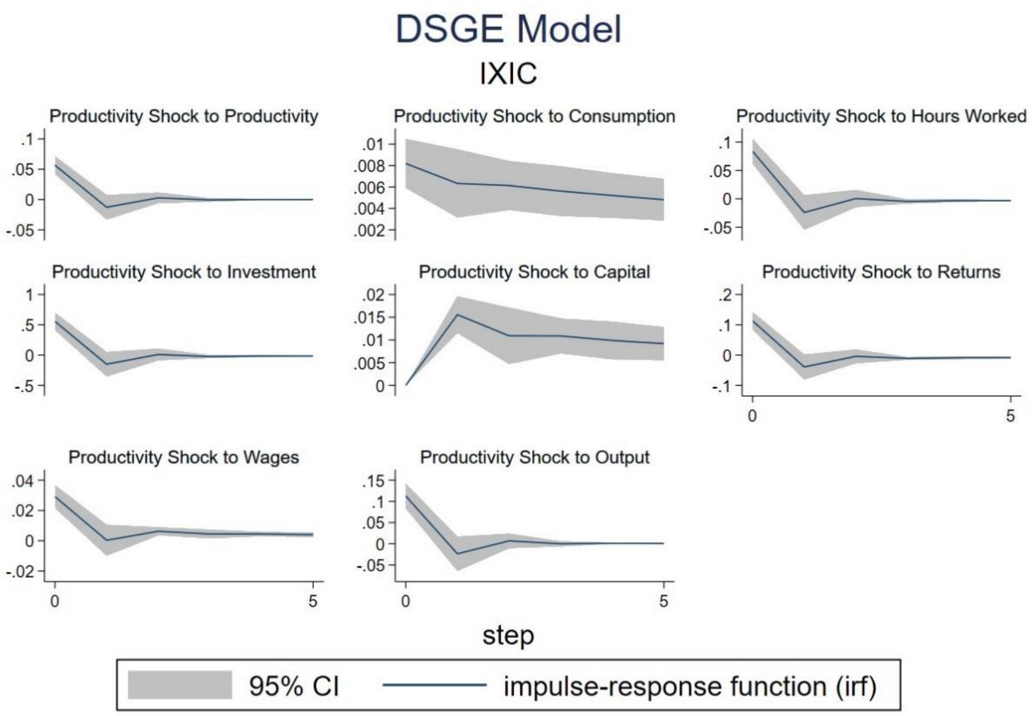


Figure 7 – DSGE Model - Nasdaq

Table 11 – Impulse Response Table IXIC

	Productivity shock to	Productivity	Productivity shock to	Productivity shock to	Productivity shock to	Productivity	Productivity	Productivity

	Productivity	shock to Output	Investment	Consumption	hours worked	shock to Returns	shock to Wages	shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.054	0.107	0.525	0.008	0.079	0.107	0.028	0
1	-0.011	-0.02	-0.132	0.006	-0.021	-0.035	0.001	0.015
2	0.002	0.006	0.004	0.006	0	-0.005	0.006	0.011
3	0	0	-0.022	0.005	-0.004	-0.01	0.004	0.01
4	0	0.001	-0.015	0.005	-0.003	-0.008	0.004	0.01
5	0	0.001	-0.015	0.005	-0.003	-0.008	0.004	0.009

Table 12 – Steady-state IXIC

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.204	0.18	-1.11	0.267	(-0.565, 0.156)

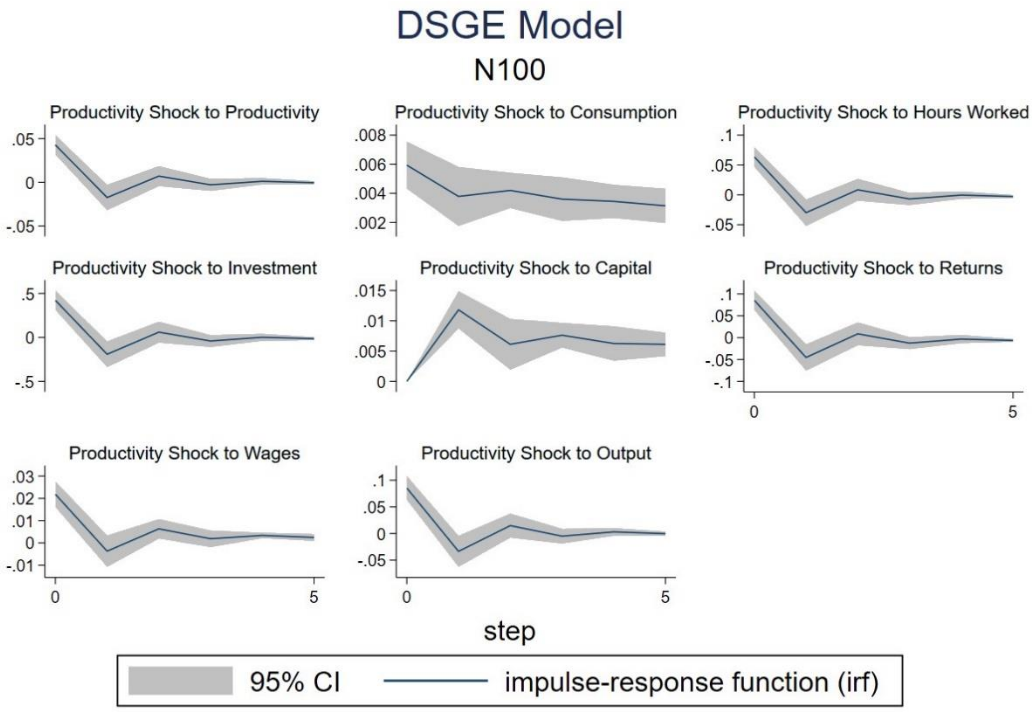


Figure 8 – DSGE Model – Euronext100

Table 13 – Impulse Response Table N100

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.04	0.08	0.395	0.006	0.059	0.08	0.02	0
1	-0.016	-0.031	-0.177	0.004	-0.028	-0.042	-0.003	0.011
2	0.006	0.014	0.054	0.004	0.008	0.008	0.006	0.006
3	-0.003	-0.004	-0.038	0.003	-0.006	-0.012	0.002	0.007
4	0.001	0.003	0	0.003	0	-0.003	0.003	0.006
5	0	0	-0.014	0.003	-0.003	-0.006	0.002	0.006

Table 14 – Steady-state N100

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.401	0.16	-2.39	0.017	(-0.73,-0.072)

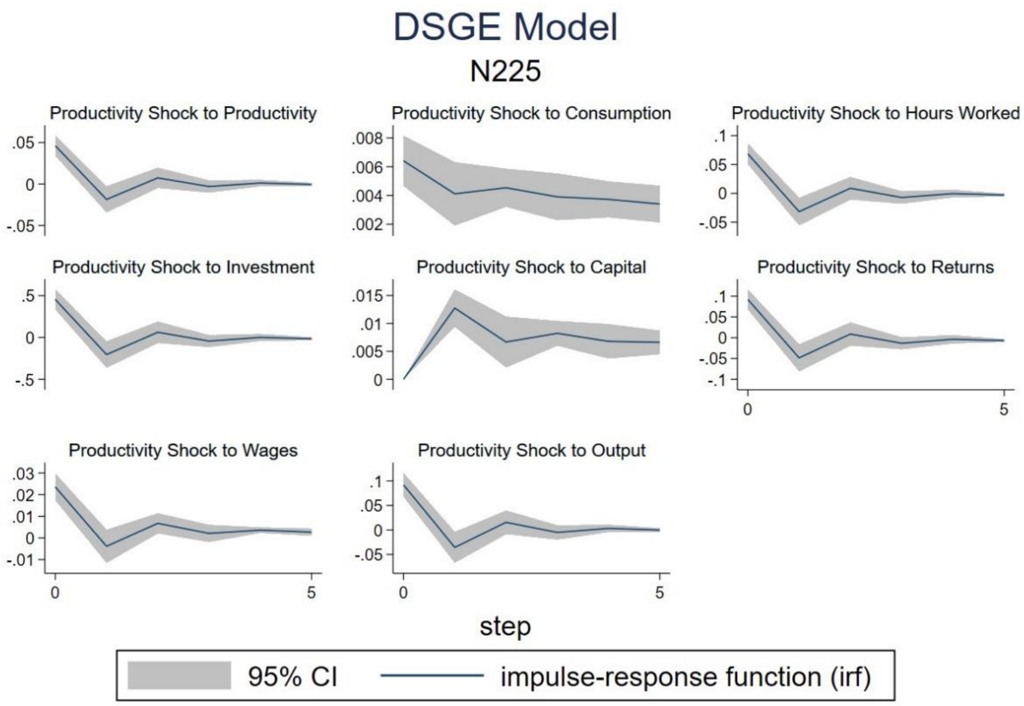


Figure 9 – DSGE Model - Nikkei

Table 15 – Impulse Response Table N225

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.043	0.086	0.427	0.006	0.064	0.086	0.022	0
1	-0.017	-0.033	-0.188	0.004	-0.029	-0.045	-0.003	0.012
2	0.007	0.014	0.056	0.004	0.008	0.008	0.006	0.006
3	-0.003	-0.004	-0.039	0.004	-0.007	-0.012	0.002	0.008
4	0.001	0.003	0	0.004	-0.001	-0.004	0.003	0.006
5	0	0	-0.014	0.003	-0.003	-0.006	0.003	0.006

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.394	0.168	-2.34	0.02	(-0.725, 0.063)

Table 16 – Steady-state N225

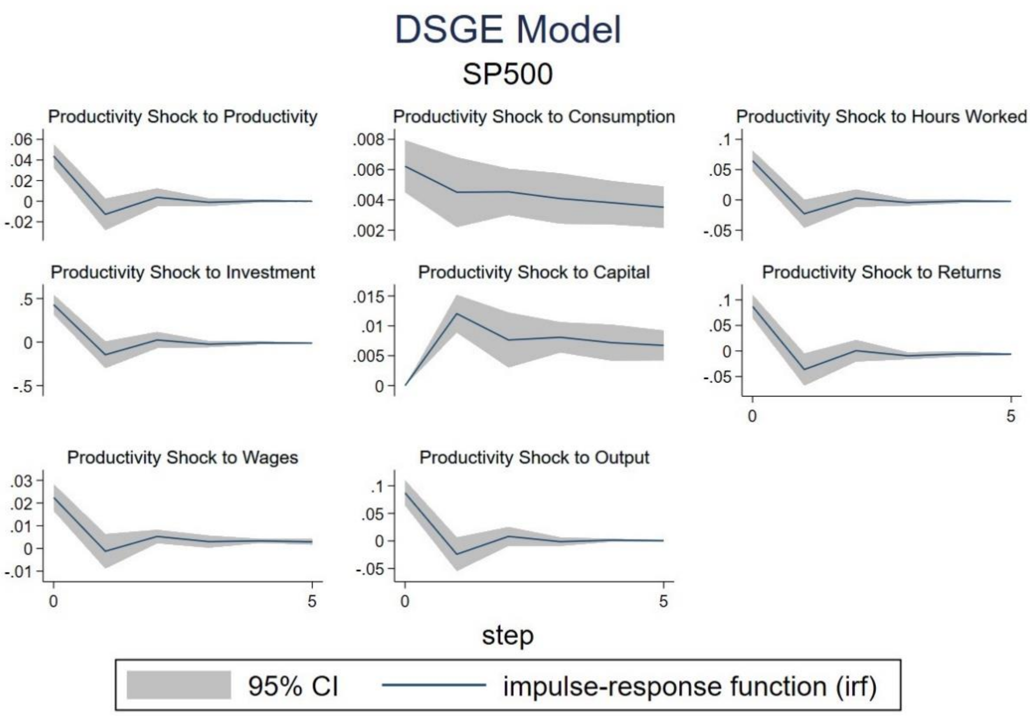


Figure 10 – DSGE Model - SP500

Table 18 – Impulse Response Table SP500

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.041	0.082	0.404	0.006	0.061	0.082	0.021	0
1	-0.012	-0.022	-0.133	0.004	-0.021	-0.033	-0.001	0.011
2	0.003	0.007	0.02	0.004	0.002	0	0.005	0.007
3	-0.001	-0.001	-0.022	0.004	-0.004	-0.009	0.003	0.008
4	0	0.001	-0.009	0.004	-0.002	-0.006	0.003	0.007
5	0	0.001	-0.011	0.003	-0.002	-0.006	0.003	0.006

Table 19 – Steady-state SP500

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval
Productivity Shock	-0.281	0.177	-1.59	0.113	(-0.63, 0.066)

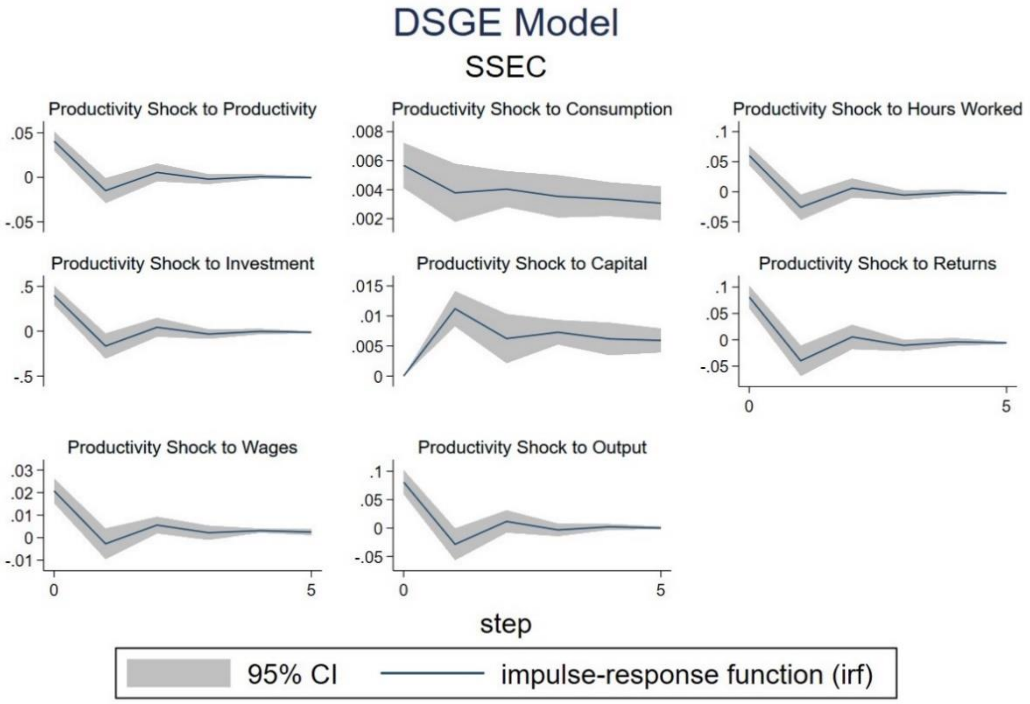


Figure 11 – DSGE Model - SSEC

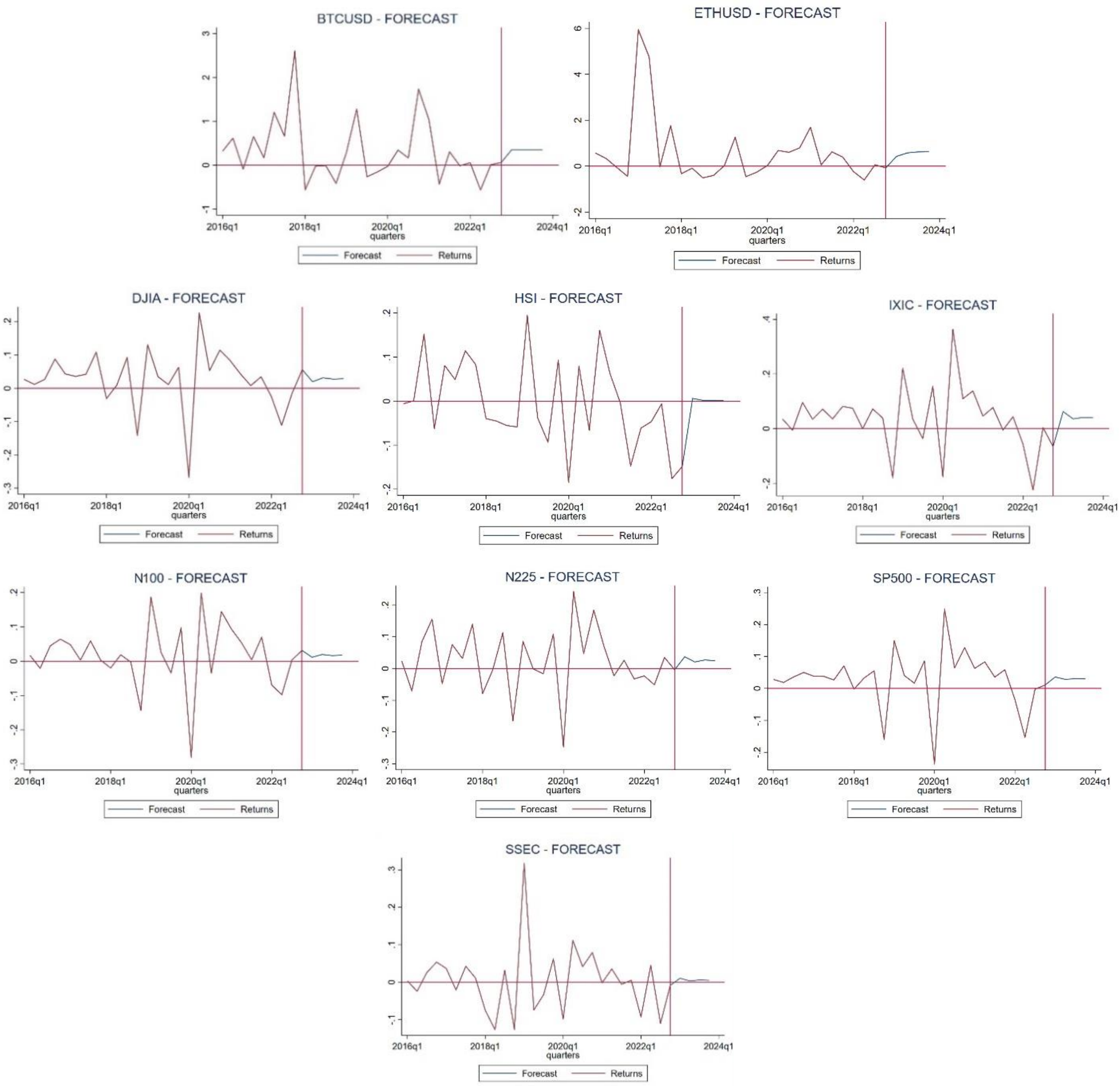
Table 20 – Impulse Response Table SSEC

	Productivity shock to Productivity	Productivity shock to Output	Productivity shock to Investment	Productivity shock to Consumption	Productivity shock to hours worked	Productivity shock to Returns	Productivity shock to Wages	Productivity shock to Capital
step	irf	irf	irf	irf	irf	irf	irf	irf
0	0.038	0.076	0.375	0.005	0.056	0.076	0.019	0
1	-0.014	-0.026	-0.153	0.004	-0.024	-0.037	-0.002	0.011
2	0.005	0.01	0.039	0.004	0.005	0.005	0.005	0.006
3	-0.002	-0.003	-0.029	0.003	-0.005	-0.01	0.002	0.007
4	0.001	0.002	-0.003	0.003	-0.001	-0.004	0.003	0.006
5	0	0	-0.012	0.003	-0.002	-0.005	0.002	0.006

Table 21 – Steady-state SSEC

Variable	Coefficient	Std. Error	z	P > |z|	95% Confidence Interval

Productivity Shock	-0.36	0.17	-2.10	0.036	(-0.67, 0.023)



Figures 13,14,15,16,17,18,19,20,21,22 are a representation of the price returns forecast of BTCUSD, ETHUSD, DJIA, HIS, IXIC, N100, N225, SP500, and SSEC

To further understand the relationship between traditional and digital economies, it is of interest to examine the interactions between the markets and determine whether traditional markets lead the digital economy, or if the digital economy leads traditional markets. By estimating the VAR model, we can examine the dynamic relationship between the variables and understand how changes in one market may influence the other. Additionally, by checking the significance of each term in the VAR model, we can identify which variables have the most impact on the system and which are the most influential in driving the relationships between the traditional and digital markets.

The VAR model we ran is a VAR(5) model with 10 equations, analyzing the relationship between traditional and digital economy markets, considering the influence of the past 5 periods' interactions. This model allows us to understand how changes in one market may influence the other, by considering the past 5 periods' interactions, this will be important to understand if traditional financial markets lead the digital economy or vice versa:

$$BTCUSD_t = +0.013 - 1.036 * N225_{t-2} - 0.692 * SSEC_{t-2} + 1.098 * N225_{t-4}$$

(46) – VAR for Bitcoin market

Equation 46: Significant influences from the Nikkei 225 and SSE Composite Index are observed in the Bitcoin market.

$$DJIA_t = +0.426 * DJIA_{t-1} + 0.269 * IXIC_{t-1} + 0.137 * N225_{t-1} - 0.435 * SP500_{t-1} + 0.107 * IXIC_{t-2} + 0.355 * N100_{t-2}$$

(47) – VAR for DJIA market

Equation 47: DJIA, IXIC, Nikkei 225, S&P 500, and EuroNext100 have substantial impacts on the DJIA market.

$$ETHUSD_t = +0.146 * BTCUSD_{t-1} - 0.714 * SSEC_{t-1} - 1.434 * N225_{t-2} + 0.171 * ETHUSD_{t-3} + 0.763 * SSEC_{t-4} + 0.164 * BTCUSD_{t-5}$$

(48) – VAR for Ethereum market

Equation 48: Bitcoin, SSE Composite Index, Nikkei 225, and Ethereum itself play important roles in determining the Ethereum market.

$$HSI_t = +0.177 * IXIC_{t-1} - 0.316 * N225_{t-1}$$

(49) – Var for Hang Seng Market

Equation 49: The IXIC and Nikkei 225 have notable influences on the Hang Seng market.

$$IXIC_t = -0.27 * N225_{t-1}$$

(50) – Var for Nasdaq Market

Equation 50: The Nikkei 225 plays a significant role in the Nasdaq market.

$$N100_t = +0.654 * DJIA_{t-1} - 0.234 * HSI_{t-1} + 0.208 * IXIC_{t-1} - 0.218 * N225_{t-1} + 0.264 * IXIC_{t-2} + 0.185 * SSEC_{t-5}$$

(51) – VAR for EuroNext100 Market

Equation 51: DJIA, IXIC, Hang Seng Index, and SSE Composite Index have considerable impacts on the EuroNext100 market.

$$N225_t = +0.713 * DJIA_{t-1} - 0.287 * HSI_{t-1} + 0.353 * IXIC_{t-1} - 0.367 * N225_{t-1} + 0.215 * SSEC_{t-2} + 0.039 * BTCUSD_{t-4} + 0.729 * DJIA_{t-4}$$

(52) – VAR for Nikkei Market

Equation 52: DJIA, IXIC, Hang Seng Index, Nikkei 225, SSE Composite Index, and Bitcoin are important factors in determining the Nikkei market.

$$SP500_t = +0.406 * DJIA_{t-1} + 0.41 * IXIC_{t-1} + 0.118 * N225_{t-1} - 0.467 * SP500_{t-1} + 0.177 * IXIC_{t-2} + 0.255 * N100_{t-2} + 0.094 * IXIC_{t-3} + 0.229 * N100_{t-4} + 0.084 * IXIC_{t-5}$$

(53) – VAR for SP500

Equation 53: DJIA, IXIC, Nikkei 225, S&P 500, and EuroNext100 have significant influences on the S&P 500 market.

$$SSEC_t = +0.171 * IXIC_{t-1} - 0.19 * N225_{t-1} - 0.151 * SSEC_{t-1}$$

(54) – Var for SSE Comp

Equation 54: The IXIC, Nikkei 225, and SSE Composite Index have notable impacts on the SSE Composite market.

It is important to note that the traditional financial markets, measured by market capitalization, are considerably larger than the digital financial market. This means that traditional financial companies have a much higher value than the capitalization of financial digital assets. This also implies that traditional finance companies may have more impact on the economy than digital financial assets. Therefore, when analyzing the relationship between traditional and digital economies, it is essential to consider this difference in market capitalization, as it could potentially skew the results of the VAR model.

For instance, the DJIA, S&P 500, Nasdaq (IXIC), EuroNext100, and Nikkei 225 are major stock market indices that represent the performance of large corporations in their respective regions. These markets tend to be closely related due to the interconnectedness of the global economy. Companies listed on these indices often have international operations, and their performances may impact other markets in various regions. Additionally, these traditional finance markets are influenced by macroeconomic factors, such as interest rates, inflation, and global trade, which can have a ripple effect on other markets.

In contrast, the digital economy markets, such as Bitcoin and Ethereum, represent the growing significance of crypto assets and respectively the adoption of blockchain technology. These markets are relatively new and have experienced rapid growth in recent years. While they are influenced by some of the same macroeconomic factors as traditional markets, their relationships with other markets may differ due to the nature of their underlying technologies and the specific dynamics of the financial digital assets markets.

The Hang Seng Index and SSE Composite Index are regional indices that reflect the performance of companies in Hong Kong and China. Similar to the major stock market indices, these regional indices are influenced by the global economy and may impact other markets within their regions. Their relationship with other markets may be affected by regional economic policies, geopolitical events, and cross-border investments.

Overall, the relationships among these various markets are complex, and their interactions can be influenced by a multitude of factors. Understanding these relationships is crucial for investors and policymakers to make informed decisions and navigate the interdependencies between traditional finance markets and the digital economy.

Conclusions

DSGE models are built on the idea of optimizing behavior, which means that individuals and firms make decisions that maximize their utility or profits. These models incorporate both microeconomic and macroeconomic concepts, such as supply and demand, inflation, and employment. One of the key features of DSGE models is their use of dynamic programming, which allows for the analysis of how economic variables change over time.

Our study was focused on analyzing the impacts of different economic policies or shocks on the markets and making predictions about how the market is likely to behave in the future. However, it is important to keep in mind that DSGE models are based on a number of assumptions and simplifications about the economy, and it is possible that the model may not accurately capture all of the relevant economic phenomena.

Our research began by utilizing the DSGE model to forecast the future of observable variables in the market. We carefully examined the interactions between variables within similar markets, interpreting them as firms. This allowed us to gain a deeper understanding of the underlying relationships and dynamics within the market.

Through our analysis, we found that the DSGE model is a reliable tool for forecasting the future of observable variables. We were able to accurately predict the behavior of various economic variables, such as productivity, investment, consumption, labor, returns, wages, and capital. This demonstrates the versatility and applicability of the model in various market contexts. Furthermore, we looked into the impact of a productivity shock on the market by analyzing the impulse-response functions of the different variables. We found that when a productivity shock occurs, all of the variables respond favorably, aligning with our utility function and the equations defined in our methodologies. This suggests that the market is able to adjust and adapt to changes in productivity, leading to an overall improvement in economic performance.

The DSGE model was used to analyze the impact of productivity shocks on different stock market indices, including Euronext100, Nikkei, SP500, and SSE Comp. The model was able to simulate the effect of productivity shocks on various variables, such as productivity, output, investment, consumption, hours worked, returns, wages, and capital. The impulse response tables for each index showed the expected pattern of responses to productivity shocks. Additionally, the p-values for each index were calculated using a z-test to determine the significance of the productivity shock on each index. If the p-value was less than 0.05, the null hypothesis was rejected, and it was concluded that the productivity shock had a significant impact on the index.

The results showed that the Euronext100, Nikkei, and SSE Comp had significant p-values of 0.017, 0.02, and 0.036, respectively, indicating that the productivity shock had a significant impact on these indices. On the other hand, the SP500 had a p-value of 0.113, which was not significant, suggesting that the productivity shock had no significant impact on this index. Furthermore, the steady-state coefficients and confidence intervals were also presented for each index.

Overall, the DSGE model was useful in analyzing the impact of productivity shocks on various stock market indices. The results highlighted that not all indices were impacted equally by productivity shocks, and some indices were more sensitive to these shocks than others. These findings could have implications for investors and policymakers who are interested in understanding the effects of productivity shocks on different stock market indices.

Regional indices, such as the Hang Seng Index and SSE Composite Index, represent the performance of companies and their relationships with other markets may be impacted by regional economic policies, geopolitical events, and cross-border investments.

Overall, the VAR model analysis helps us understand the dynamic relationship between traditional and digital finance markets and how they interact with each other. While traditional markets may have a more significant impact on the economy, digital markets are growing in importance and will continue to play an essential role in the global economy in the years to come.

Our findings indicated that digital assets are starting to play a significant role in market movements, although their impact is not as significant as traditional markets, due to their smaller market capitalization. There are several potential impact shocks that could originate from traditional markets and influence crypto markets. We can note that some major financial events like an economic recession, could have a downturn in the global economy which could lead to a decreasing demand for financial digital assets, as investors look to more stable assets in these types of scenarios. Interest rate changes can also play an important role, as the financial digital assets valuations have been known to be sensitive to changes in interest rates, affecting the cost of borrowing and the opportunity cost of holding non-yielding assets like crypto assets.

Regulatory changes are also a factor worth mentioning, as the changes in government regulations and laws surrounding crypto assets could greatly impact their adoption and value. Geopolitical events seem to capture the recent headlines as political instability and events such as war or natural disasters could lead to increased demand for decentralized and borderless assets like crypto assets.

Overall, we have seen that the markets tend to follow the lead of the major stock markets such as the S&P 500, Nikkei, and Dow Jones Industrial Average. Additionally, we observed that Bitcoin and Ethereum appear to have their autoregressive process, and it was interesting to note that Bitcoin also appears as a cause in the Nikkei market.

The VAR models in the study reveal that there are significant relationships between different global stock markets. For example, the DJIA, Nasdaq, Hang Seng, Nikkei 225, S&P 500, and EuroNext100 markets all exhibit interconnections with each other. The coefficients in the equations suggest that lagged values of these markets can impact the current values of each other, indicating the presence of dynamic relationships between them.

Additionally, the study finds that crypto assets, such as Bitcoin and Ethereum, also play important roles in determining the values of other financial assets. For example, the VAR model for the Ethereum market shows that the lagged values of Bitcoin, SSE Composite Index, Nikkei 225, and Ethereum itself can influence the current value of the Ethereum market.

These findings suggest that investors and analysts should consider the interconnections between these markets when making investment decisions. Diversification across different markets and asset classes may help investors reduce their exposure to the risks associated with these interconnections.

Our research indicates that digital markets are starting to play an increasingly important role in the global economy, and their influence will likely continue to grow in the future. It is also important to note that the results are skewed by traditional markets having much higher market capitalization, but digital assets are still showing to have a significant impact on the global economy.

REFERENCES

Asimakopoulos, S., Lorusso, M., & Ravazzolo, F. (2019). A New Economic Framework: A DSGE Model with Cryptocurrency. Centre for Applied Macroeconomics and Commodity Prices, *CAMP Working Paper Series*, No7/2019

Blanchard, O. (2018). On the future of macroeconomic models. *Oxford Review of Economic Policy*, Volume 34, Numbers 1–2, 2018, pp. 43–54, https://doi.org/10.1093/oxrep/grx045

Calvo, G. A., (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3), pp.383-398, https://doi.org/10.1016/0304-3932(83)90060-0

Christiano, L. J., Eichenbaum M., & Evans C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, 113(1), pp.1-45, https://doi.org/10.1086/426038

Christiano, L. J., Eichenbaum, M. S., & Trabandt, M. (2018). On DSGE Models, *Journal of Economic Perspectives*, 32, pp.113–40, https://doi.org/10.1257/jep.32.3.11

Clarida, R., Gali, J., & Gertler, M. (1999). The science of monetary policy: a new Keynesian perspective. *Journal of economic literature*, 37(4), pp. 1661-1707, https://doi.org/10.1257/jel.37.4.1661

Clemens, Sialm (2006). Stochastic taxation and asset pricing in dynamic general equilibrium. *Journal of Economic Dynamics and Control*, Volume 30, Issue 3, pp.511-540, ISSN 0165-1889, Economy, ,91(1), pp.39–69, https://doi.org/10.1016/j.jedc.2005.02.004

Friedman, M., (1968). The role of monetary policy, *American Economic Review*, 58 (1), pp. 1–17

Gali, J., David Lopez-Salido, J., and Valles, J. (2007). Understanding the Effects of Government Spending on Consumption, *Journal of the European Economic Association*, Vol 5(1): pp. 227–70, https://doi.org/10.1162/JEEA.2007.5.1.227

Kolasa, M., Rubaszek, M., & Skrzypczyński P. (2012). Putting the New Keynesian DSGE Model to the Real-Time Forecasting Test. *Journal of Money, Credit and Banking*, 44(7), pp.1301-1324, https://doi.org/10.1111/j.1538-4616.2012.00533.x

Kydland, Finn E., & Edward, C. Prescott, (1982). Time to Build and Aggregate Fluctuations, *Econometrica*, pp. 1345–1370

Liu, A., Song, H., & Blake, A. (2018). Modelling productivity shocks and economic growth using the Bayesian dynamic stochastic general equilibrium approach, *International Journal of Contemporary Hospitality Management*, Vol. 30 No. 11, pp. 3229-3249, https://doi.org/10.1108/IJCHM-10-2017-0686

Long, J. B. & Plosser C.I. (1983). Real Business Cycles, *Journal of political.*

Lucas, R. E., (1978). Asset Prices in an Exchange Economy. *Econometrica*, 46, pp.1429–1445, http://www.jstor.org/stable/1913837

Nelson, E. (2002). Direct effects of base money on aggregate demand: theory and evidence, *Journal of Monetary Economics*, 49(4), pp.687-708

Olatunji, A. S., Oladimeji, T. S. (2019). Carbon policy for the United States, China and Nigeria: An estimated dynamic stochastic general equilibrium model, *Science of The Total Environment*, Volume 697, https://doi.org/10.1016/j.scitotenv.2019.134130

Smets, F., & Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, *Journal of the European Economic Association*, 1(5), pp.1123-1175, https://doi.org/10.1162/154247603770383415

Stiglitz, J.E. (2018). Where modern macroeconomics went wrong. *Oxford Review of Economic Policy*, Volume 34, Numbers 1–2, 2018, pp. 70–106, https://doi.org/10.1093/oxrep/grx057

Woodford, M. (2003). Optimal interest-rate smoothing. *The Review of Economic Studies*, 70(4), pp. 861-886, http://www.jstor.org/stable/3648627

Yang, Y., Zhang, H., & Chen, X. (2020). Coronavirus pandemic and tourism: Dynamic stochastic general equilibrium modeling of infectious disease outbreak. *Annals of tourism research*, 83, 102913, https://doi.org/10.1016/j.annals.2020.102913

Yun, T. (1996). Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles. *Journal of Monetary Economics*, 1996, 37 (2), pp.345–370, https://doi.org/10.1016/S0304-3932(96)90040-9