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INFLATION AND GROWTH WITH THE MIU APPROACH AND THE EQUATION OF EXCHANGE

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Abstract

The purpose of this paper is to study a relationship between growth and inflation on the basis of the Tobin growth model. We deviate from the Tobin approach by determining money demand with the money-in-utility (MIU) approach. The utility is affected by money holding. The utility function is applied by an alternative approach proposed by Zhang. The wealth accumulation a key determinant of economic growth. The government supplies money which is described by the traditional equation of exchange. The velocity of money is determined as a function of the rate of interest as in the Baumol-Tobin model. We build the dynamic model and simulate the motion of the model. We carry out comparative dynamic analysis in various parameters.

Keywords: inflation; Tobin growth model; equation of exchange; Baumol-Tobin model;MIU approach.

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1 Introduction

Relations between inflation and growth have been a key issue in theoretical and empirical economics. As with regards to relationship between almost any two closely related variables at any point of time. In a recent comprehensive review on the literature of the relationship between economic growth and inflation in developed and developing economies, Akinsola and Odhiambo (2017) show that researchers find varied relations between inflation and growth. They list up four conclusions from the literature: i) inflation does not have any impact on economic growth (e.g., Sidrauski, 1967a; and Cameron et al., 1996); ii) inflation is positively related to economic growth (e.g., e.g., Benhabib and Spiegel, 2009); iii) inflation is negatively related to economic growth (e.g., e.g., Friedman, 1956; Stockman, 1981; Fischer, 1983); iv) inflation affects economic growth in terms of specific thresholds (e.g., Aydin et al., 2016). The purpose of this study is to address issues related to relations between growth and inflation by building a monetary growth model with microeconomic foundation. The model is based on the quantity theory of money, the money in utility approach, and neoclassical growth theory.

This study is concerned with dynamic interdependence between growth, money, and inflation within neoclassical growth framework. The seminal contribution in the theory of monetary growth within the framework of neoclassical growth theory was published by Tobin (1965). He studies an isolated economy in which the outside moneyissued by the governmentcompetes with real capital in the portfolios of agents within the framework of the Solow model. The real sector is the same as in the Solow growth model. Nevertheless, money demand in the Tobin model is not built on microeconomic foundation. An approach to money demand with microeconomic foundation is the so-called money in utility (MIU) function approach. In this approach money yields some services and just directly enters into the utility function (Eden, 2005: Chap. 2). The approach was applied initially by Patinkin (1965), Sidrauski (1967) and Friedman (1969). Since then economists apply the approach to address various issues related to money and inflation. Sidrauski (1967a) challenged Tobin's non-neutrality result. In his specified framework, he found that money is superneutral in steady state and changes in the inflation rate have no effect on all the real variables in the economy. Wang and Yip (1992) show that Sidrauski'ssuperneutralityis invalid if leisure is introduced into the utility function.There are many other issues and models related to interactions between monetary policy and economic growth (e.g.,Feenstra, 1986;Gomme, 1993; van der Ploeg and Alogoskoufits, 1994; Jones and Manuelli, 1995; Dotsey and Starte, 2000; Chappell and Matthews, 2001; Meng and Yip, 2004; and Handa, 2009; Burdett *et al*.2017; Araujo and Hu, 2018; Boel, 2018; and Kraft and Weiss, 2019).This study is based on the MIU approach. But we deviate from the literature in that we use an alternative utility function to modeling household behavior. The rest of the paper is organized as follows. Section 2 defines the monetary growth model with wealth accumulation. Section 3 identifies the two differential equations for describing movement of the system and simulates the model. Section 4 carries out comparative dynamic analysis with regards some parameters. Section 5 concludes the study.

2 The monetary growth model with education

Many aspects of this model are similar to those in the Tobin monetary growth model (Tobin, 1965; Nagatani, 1970), except that the money demand and supply are determined with mechanisms different from those used in the Tobin model. In describing economic production, we follow neoclassical growth theory (e.g., Solow, 1956; Uzawa, 1961; Burmeister and Dobell, 1970;Azariadis, 1993; Barro and Sala-i-Martin, 1995).The economy produces a homogenous commodity. Firms use capital and labor as input factors. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. The economy has two assets, money and capital stock. The household may hold two assets.

Labor supply

There is a homogenous population denoted by$\bar{N}$.Let $T(t)$stand for the work time of a representative household and $N(t)$ for the flow of qualified labor services used at time t for production. We have$N(t)$ as follows:

$$N(t) = T(t)\bar{N}. \quad (1)$$

Production sector

We use a conventional production function to describe a relationship between inputs and output. There two production factors, capital $K(t)$and labor $N(t)$, which are assumed to be fully employed. We use the following production function $F(t)$to describe a relationship between inputs and output:

$$F(t) = AK^{\alpha}(t)N^{\beta}(t), \alpha, \beta > 0, \alpha + \beta = 1, (2)$$

in which $A, \alpha,$ and β are positive parameters. Markets are competitive; thus labor and capital earn their marginal products. We neglect possible money input in the production function. This implies that economic growth is attained by labor and capital (e.g., Snowdon and Vane, 2005). The rate of interest $r(t)$ and wage rate$w(t)$ are determined by markets. Hence, for any individual firm $r(t)$ and $w(t)$ are given at each point of time. The production sector chooses the two variables $K(t)$ and $N(t)$ to maximize its profit.The marginal conditions are given by:

$$r(t) + \delta_k = \frac{\alpha\, F(t)}{K(t)}, \;\; w(t) = \frac{\beta\, F(t)}{N(t)}, \;\; (3)$$

where δ_k is the fixed depreciation rate of physical capital.

Disposable income

We apply the concept of disposable income and utility proposed by Zhang (1993, 2005) Consumers make decisions on choice of consumption level of commodity, saving, and money holding. In this study, we follow Zhang (2008) in modeling choice of money. The preference over current and future consumption is reflected in the consumer's preference structure over education, money, consumption and saving.Money is introduced by assuming that a central bank distributes at no cost to the population a per capita amount of fiat money $M(t) > 0$.Let$P(t)$stands for the price of money. The scheme according to which the money stock evolves over time is deterministic and known to all agents.The government (positive or negative) expenditure in real terms per capita, $\tau(t)$,is given by:

$$\tau(t) = \frac{\dot{M}(t)}{P(t)}.$$

We will provide a mechanism to determine $\dot{M}(t)$.Per household current income from the interest payment $r(t)\bar{k}(t)$,the wage payments$T(t)w(t)$,the cost of holding money $\pi(t)\, m(t)$, and the income from government $\tau(t)$is given by:

$$y(t) = r(t)\bar{k}(t) + T(t)\, w(t) - \pi(t)\, m(t) + \tau(t), \quad (4)$$

where $\pi(t)$ is the inflation rate and $m(t) \equiv M(t)/P(t)$. The wage income is given by $W(t) = T(t)\, w(t)$. The total value of wealth of the representative household is$a(t)$where:

$$a(t) \equiv \bar{k}(t) + m(t).$$

Here, we do not allow borrowing for current consumption. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The disposable income of a household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving, $\hat{y}(t) = y(t) + a(t)$.That is:

$$\hat{y}(t) = \bar{k}(t) + m(t) + r(t)\bar{k}(t) + T(t)\, w(t) - \pi(t)m(t) + \tau(t). \quad (5)$$

The disposable income is used for saving, consumption, and money holding.

Denote $\bar{T}(t)$ the time spent on leisure. Let the (fixed) total available time be denoted by T_0. The time constraint is expressed by:

$$T(t) + \bar{T}(t) = T_0. (6)$$

Insert (6) in (5)

$$\hat{y}(t) = \bar{y}(t) + m(t) - \pi(t)\, m(t) - \bar{T}(t)\, w(t), \quad (7)$$

where

$$\bar{y}(t) \equiv \bar{k}(t) + r(t)\bar{k}(t) + T_0\, w(t) + \tau(t).$$

Utility function and optimal behavior

The household's utility function for enjoying leisure, holding money, consuming goods, and making saving is be represented by the following utility function:

$$U(t) = \bar{T}^{\sigma_0}(t) m^{\varepsilon_0}(t) c^{\xi_0}(t)\, s^{\lambda_0}(t), \ \ \sigma_0,\ \varepsilon_0,\ \xi_0, \lambda_0 > \ 0, \ \ (8)$$

whereσ_0 is the propensity to enjoy leisure time,ε_0 is propensity to hold money ξ_0 the propensity to consume, and λ_0 the propensity to own wealth.This utility function is applied to different economic problems. A detailed explanation of the approach and its applications to different problems of economic dynamics are provided in Zhang (2005, 2008).

The disposable income is spent on holding money, consumption of the good, and saving. We have:

$$\big(1 + r(t)\big)\, m(t) + c(t) + s(t) = \hat{y}(t). \ (9)$$

Insert (7) in (9)

$$w(t)\bar{T}(t) + \bar{\pi}(t)\, m(t) + c(t) + s(t) = \bar{y}(t), \ \ (10)$$

where

$$\bar{\pi}(t) \equiv \pi(t) + r(t).$$

Here, $\bar{\pi}(t)$is the opportunity cost of holding money. The consumer problem is to choose current money, leisure time, consumption, and saving so that the utility is maximized. Maximizing $U(t)$ subject to (10) yields:

$$w(t)\bar{T}(t) = \sigma\, \bar{y}(t), \ \ \bar{\pi}(t)\, m(t) = \varepsilon\, \bar{y}(t), \ \ c(t) = \xi\, \bar{y}(t), \ \ s(t) = \lambda\, \bar{y}(t), \ \ (11)$$

where

$$\sigma \equiv \rho\,\sigma_0,\;\; \varepsilon \equiv \rho\varepsilon_0,\;\; \xi \equiv \rho\,\xi_0,\;\; \lambda \equiv \rho\,\lambda_0,\;\; \rho \equiv \frac{1}{\varepsilon_0 + \xi_0 + \lambda_0 + \sigma_0}.$$

It should be noted that in the well-known Baumol-Tobin model (Baumol, 1952, Tobin, 1956; Romer, 1986), the demand for money is given by:

$$\frac{M}{P} \equiv \left(\frac{C\,Y}{2\,r}\right)^{1/2},$$

where C is the fixed transaction cost per transfer and Y is disposable income.Our demand function is quite similar to the Baumol-Tobin model, even though they are derived from different mechanisms.

Wealth dynamics

The change in wealth is saving minus dissaving:

$$\dot{a}(t) = s(t) - a(t).\quad (12)$$

Money supply

We use $\mu(t)$to stand for the growth rate of the money stock $M(t)$:

$$\mu(t) = \frac{\dot{M}(t)}{M(t)}.$$

In the literature of monetary economic growth theory, it is traditionally assumed that μ is a positive parameter, called inflation policy. The variable $\mu(t)$ is decided by the government. The government expenditure in real terms per capita $\tau(t)$now becomes:

$$\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{\mu(t)\, M(t)}{P(t)} = \mu(t)\, m(t). \quad (13)$$

The representative household receives (or is taxed away) $\mu(t)\, m(t)$ units of money from the government.

From $M(t) = P(t)\, m(t)$, we have

$$\pi(t) = \frac{\dot{P}(t)}{P(t)} = \mu(t) - \frac{\dot{m}(t)}{m(t)}. \quad (14)$$

The quantity theory of money assumes that the general price level of goods and services is proportional to the money in circulation. Being influenced by this theory, we assume that money supply is determined so that the following equation is satisfied:

$$V(t)\, M(t) = P(t) c(t). \quad (15)$$

where $V(t)$ is the velocity of money. For a moment, we just consider $V(t)$ a time-dependent variable. We will consider it a function of the rate of interest. The velocity of money is a market-determined variable, which is influenced by many factors, such as inflation rate, market sizes, and technologies of transaction (Anderson, et al., 2017). The government supplies the money which the equation of exchange is satisfied. This is a simplified behavior of the government's monetary policy. There are other rules to determine behavior of the government's monetary policy. For instance, the government might directly decide the money supply by optimizing inflation rate (e.g., Finocchiaro, 2018; and Oikawa andUeda, 2018).

We have thus built the dynamic model. We now examine its dynamics.

3The Dynamics of the Economic System

We first show that the dynamics are determined by two differential equations. We introduce a variable as follows:

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}.$$

The following lemma is proved in the appendix.

Lemma

The motion of the economic system is described by the following two differential equations with $m(t)$ and $z(t)$ as the variables:

$$\dot{m}(t) = m(t)\varphi_m\big(z(t), m(t)\big),$$
$$\dot{z}(t) = \varphi_z\big(z(t), m(t)\big), \quad (16)$$

where φ_m and $\varphi_z$are functions of$m(t)$ and $z(t)$ defined in the Appendix. All the other variables are determined as functions of $m(t)$ and $z(t)$ by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow \bar{k}(t)$ by (A15) $\rightarrow K(t) = \bar{k}(t)\bar{N} \rightarrow M(t)$ by (A6) $\rightarrow P(t)$ by (A8) $\rightarrow \bar{y}(t)$ by (A4) $\rightarrow F(t)$ by (A3) $\rightarrow N(t)$ by (A1) $\rightarrow T(t)$

by (1) $\rightarrow \pi(t)$ by (A8) $\rightarrow c(t),\ s(t)$, and $\bar{T}(t)$by (13) $\rightarrow \mu(t)$ by (A6) $\rightarrow \tau(t)$ by (A13).

TheLemma is important as it tells us how to follow the motion of the economic system, given proper initial conditions. With computer it is straightforward to reveal the motion of the dynamic economic system. As the expressions are too tedious, we cannot easily explicitly interpret the analytical results. For illustration, we simulate the model.We specify the velocity of money as follows:

$$V(t) = V_0 e^{a_0\, r(t)},$$

where $V_0 > 0$ and a_0 are parameters. The specification implies that if $a_0 > 0$,then the velocity is positively related to the rate of interest. A positive relation between the velocity and rate of interest is derived by Baumol (1952) and Tobin (1956).

We specify the parameter values as follows:

$$\bar{N} = 100, T_0 = 24, \alpha = 0.35, A = 1.5, V_0 = 1, a_0 = 2, \lambda_0 = 0.8, \xi_0 = 0.1, \sigma_0 = 0.2,$$

$$\varepsilon_0 = 0.01, \delta_k = 0.05.$$

The population is 100.The total available time is 24. The velocity parameter is 2. The propensity to save is 0.8.The propensities to consume goods and use leisure time are respectively 0.1 and 0.2. The propensity to hold money is 0.01. We demonstrate that with the above specified parameters, the system has a unique equilibrium point. The equilibrium values of the variables are as follows:

$$F = 3519.3, N = 1024.9, K = 10923.3, \mathrm{V} = 1.134, \mathrm{w} = 2.23, \mathrm{r} = 0.063, \pi = 0.051,$$

$$m = 13.54\ , \bar{k} = 109.2\ , a = 122{,}8, c = 15.35, T = 10.25.$$

The long-run inflation rate is 5.1 percent. This also implies that if the government increases money supply at 5.1 percent in steady state, the system will remain stationary. Thetwo eigenvalues are:

$$\{-160.9,\ \ 0\}.$$

The equilibrium point is neutral. We specify the following initial conditions:

$$z(0) = 0.05,\ \ m(0) = 13.3.$$

The changes of the variables over time are plotted in Figure 1. The system becomes stationary in the long term. The national output falls over time from the initial state. Similarly, the other variable variables fall, except that the consumption level rises slightly over time. The velocity of money rises in association with rises in the rate of interest. The inflation rate falls. The money change rate is negative initially and positive in the long term.

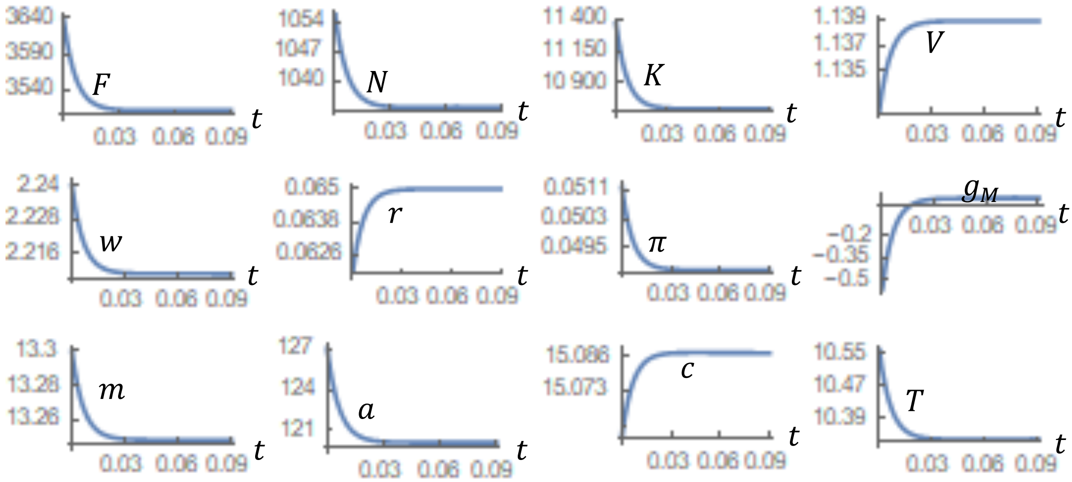

Figure 1. The Motion of the System with Money and Division of Labor

4. Comparative DynamicAnalysis

The previous section identified the equilibrium point of the dynamic economy and demonstrates that the economic system is neutral. This section examines impact of changes in some parameters on the dynamics of the system. First, we introduce a symbol $\bar{\Delta}$ to stand for the change rate in term of percentage due to the parameter change.

4.1. The velocity of money is more strongly affected by the rate of interest

We now study what happen to the economic system if the velocity of money is more strongly affected by the rate of interestas follows: $a_0 = 2$ to 2.5. The simulation result is given in Figure 2. The change values are compared with the values of the corresponding variables in Figure 1. The velocity is increased due to the change in how the rate of interest affects the velocity. The inflation rate is increased. The money change rate is increased (comparing the corresponding value of g_M in Figure 1). The household works less hours and the national labor supply falls. The national capital and national output fall initially and rise in the long term. The wage rate is increased. The household holds more real money. The household has less wealth initially and more in the long term. The consumption level rises. We conclude that if the household's money holding more strongly reacts to the rate of interest, the macroeconomic real variables and household's wealth and consumption are improved in the long term, even though the short-run reactions are positive for some variables and negative for other variables.

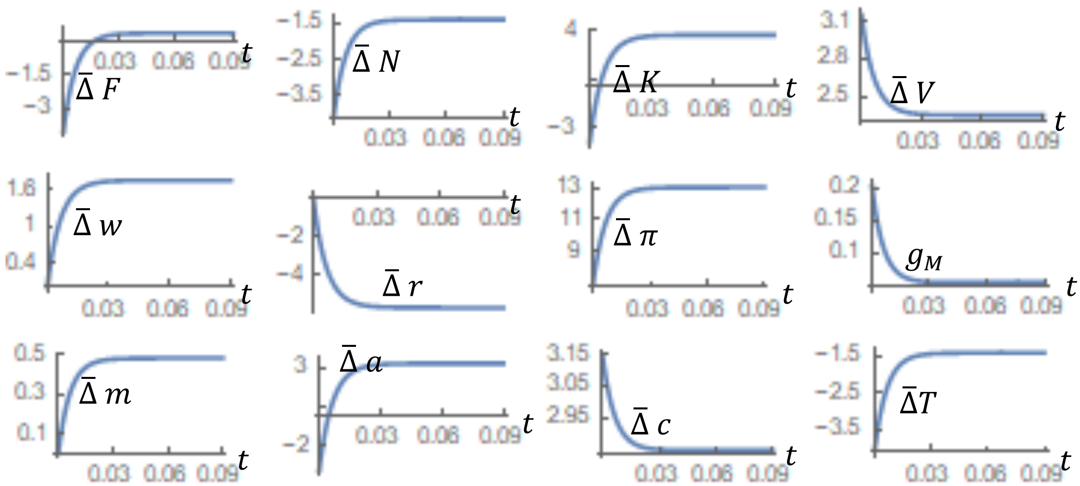

Figure 2. The Velocity of Money is More Strongly Affected by the Rate of Interest

4.2. The total factor productivity is enhanced

We now examine the impact of the following technological change: $A = 1.5$ to 1.55.The simulation result is given in Figure 3. The national output is increased. The household works more hours and the national labor supply is increased. The national capital is increased initially but is reduced slightly in the long term. The household has more wealth initially and less in the long term. The household holds less real money. The rate of interest and velocity of money are increased. The change rate of money is slightly faster. The wage rate is increased.

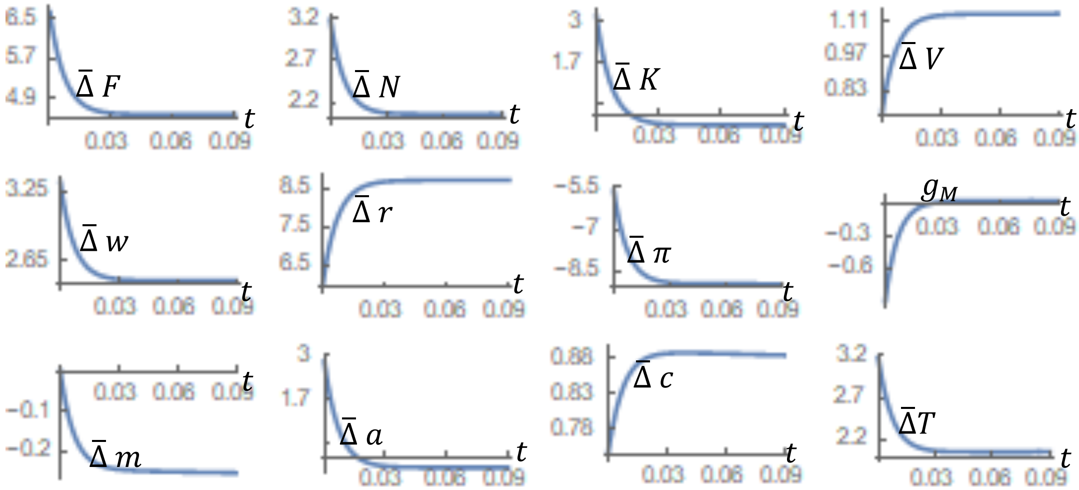

Figure 3.The Total Factor Productivity is Enhanced

4.3. The propensity to hold money is increased

We now examine the impact of the following change in the propensity to hold money: $\epsilon 0 = 0.01$ to 0.011.The simulation result is given in Figure 4. We see that as the household desires to hold more money (with the given disposable income), the real variables in the economic system are affected, but the inflation rate and the change rate of money are changed. The wage rate, rate of interest rate, and the velocity of money are invariant.

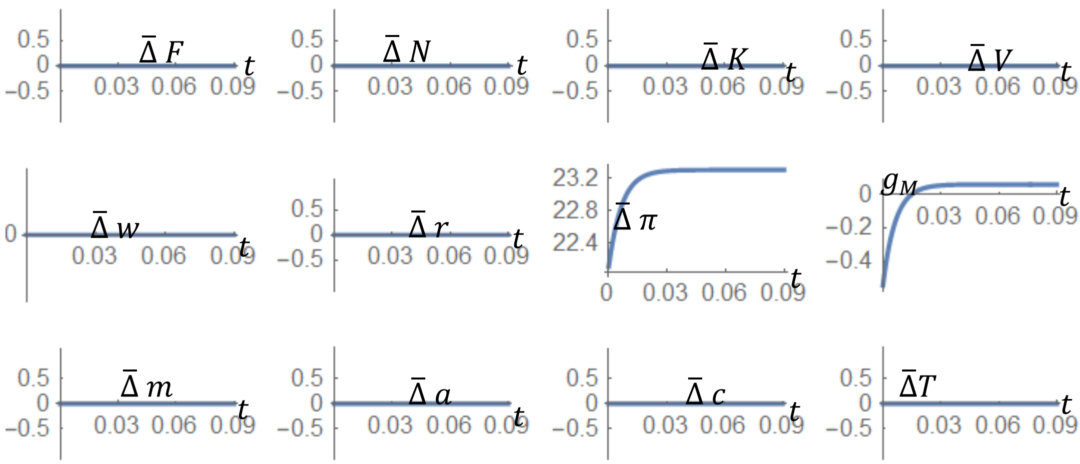

Figure 4.The Propensity to Hold Money is Increased

4.4. The propensity to save is enhanced

We now examine the impact of the following rise in the propensity to save: $\lambda 0 = 0.08$ to 0.81.The simulation result is given in Figure 5. The household has more wealth. The national capital stock is enhanced. The national output is increased. The household works more hours and the national labor supply is increased. The household holds more real money and consume less. The rate of interest falls and velocity of money is reduced. The inflation rate is increased. The wage rate is increased. The money change rate is increased.

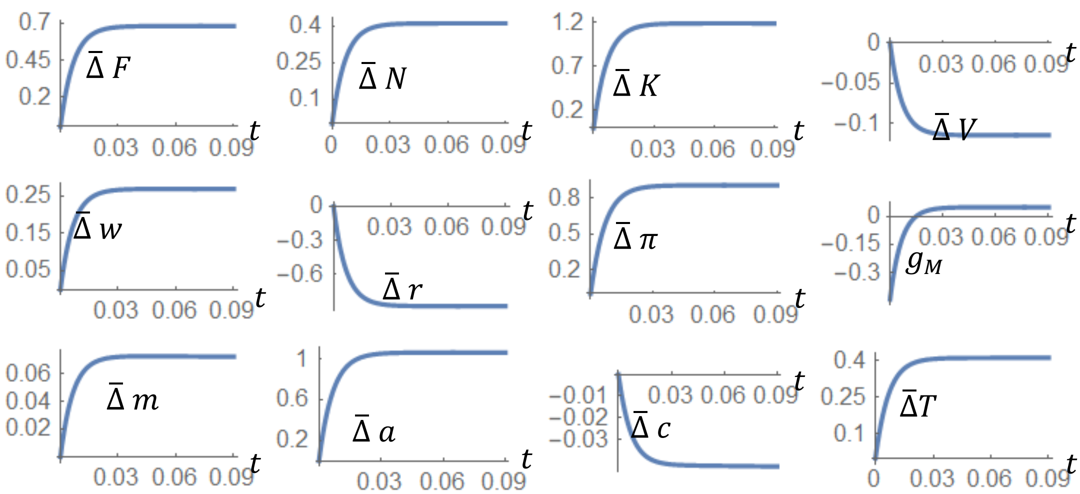

Figure 5.The Propensity to Save is Enhanced

4.5. The depreciation rate of physical capital is increased

We now study what happen to the economic system if the depreciation rate of physical capital is increased as follows: $\delta_k = 0.05$ to 0.055.The simulation result is given in Figure 6. The national capital rises initially and falls in the long term. The household initially works more hours but does not change the work time in the long term. The national labor supply is not affected in the long term. The output is increased initially and enhanced slightly in the long term. The inflation is increased. The rate of interest and the velocity of money are reduced. The wage rate falls. The household has more wealth initially and less in the long term. The consumption level of the household is reduced. The household holds less money.

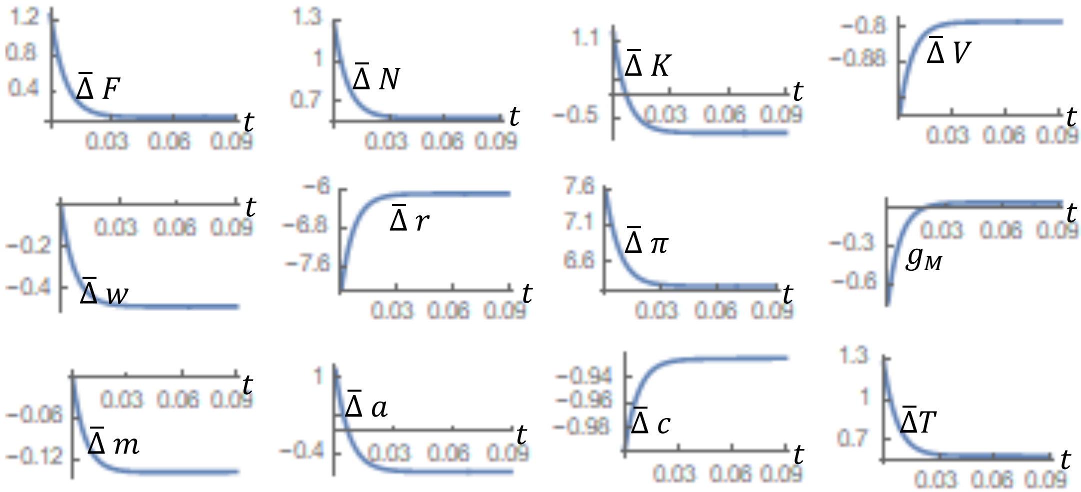

Figure 6.The depreciation rate of Physical Capital is Increased

4.6. The output elasticity of physical capital is increased

We now study what happen to the economic system ifthe output elasticity of physical capital is increased as follows: $\alpha = 0.35$ to 0.36.The simulation result is given in Figure 7. The household initially works more hours. The national labor supply is increased. The national capital rises. The output is increased. The inflation rate is reduced. The rate of interest and the velocity of money are increased. The wage rate rises. The household has more wealth and consumes more. The household holds less money.

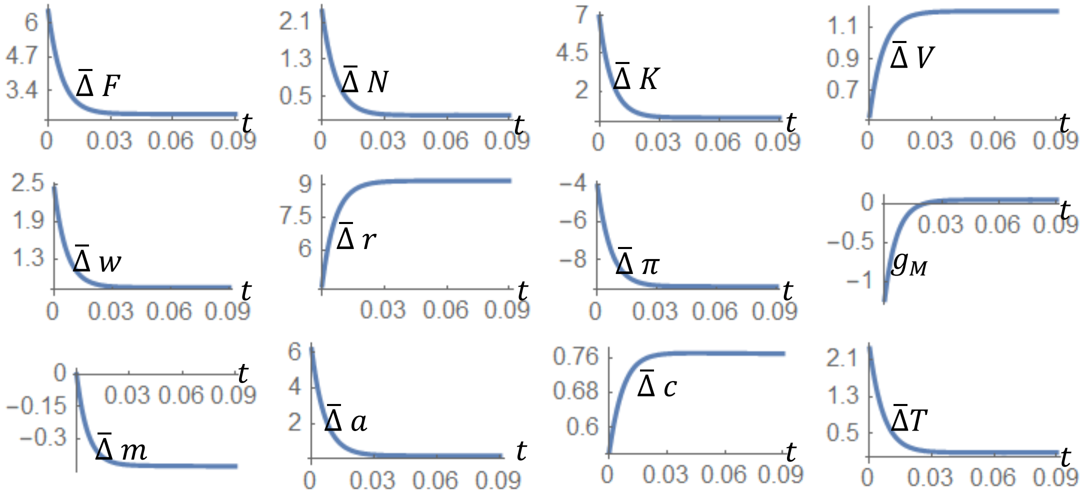

Figure 7.The Output Elasticity of Physical Capital is Increased

5.Conclusions

Following the well-known Tobin model, we introduce money in the individual saving portfolio and as medium of exchange to neoclassical growth theory. We built the monetary growth model with microeconomic foundation.Our model is based on on the quantity theory of money, the money in utility approach, and neoclassical growth theory. The wealth accumulation is the key determinant of economic growth like in neoclassical growth theory. Money is introduced to neoclassical growth model by assuming that the utility is affected by money holding. The government supplies money which is described by the traditional equation of exchange. The velocity of money is determined as a function of the rate of interest as in the Baumol-Tobin model. We first built the dynamic model and then simulated the model. We also carried out comparative dynamic analysis in various parameters. Our comparative analysis provided some insights into relations between growth and inflation over the whole dynamic process rather than only with regards to the steady state as in most of the theoretical literature of monetary growth. For instance, after comprehensively and extensively reviewing the literature of both empirical and theoretical researches on growth and inflation, Akinsola and Odhiambo (2017) show that there is an "overwhelming support in favour of a negative relationship between inflation and growth, especially in developed economies." From the figures of the comparative dynamic analysis we see that different relations between inflation and growth are possible. Our conclusion hints on why there are some different opinions on relationships between inflation and growth. It is well known that one-sector growth model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines in the literature. It is straightforward to develop the model in discrete time. We may analyze behavior of the model with other forms of production or utility functions. It is not difficult to integrate the ideas in this paper into a multi-regional economy with the Taylor rule (Zhang, 2017, 2019).

Appendix: Proving the Lemma

From (3), we obtain:

$$z \equiv \frac{r + \delta_k}{w} = \frac{\bar{\beta} \, N}{K}, \quad (A1)$$

where $\bar{\beta} \equiv \alpha / \beta$ and the time index is suppressed wherever no confusion. From (A1) and (3), we obtain:

$$w = \beta \, A \left(\frac{\bar{\beta}}{z} \right)^{\alpha}, \quad r = z \, w \, - \, \delta_k. \quad (A2)$$

We note that r and w are uniquely determined as functions of z by (A2). From (3) the definition of $\bar{y}$, we have

$$\mathrm{F} = \frac{w\,N}{\beta}. \quad \text{(A3)}$$

From (7) we have:

$$\bar{y} = R\,\bar{k} + T_0\,w + m\,\frac{\dot{M}}{M}, \quad \text{(A4)}$$

where $\mathrm{R} \equiv 1 + r$.From (15) and (11), we have:

$$\frac{V\,M}{\xi\,P} = \bar{y}. \,(A5)$$

From (A4) and (A5), we have:

$$\frac{\dot{M}}{M} = \frac{V}{\xi} - \frac{R\,\bar{k}}{m} - \frac{T_0\,w}{m}. \quad (A6)$$

From (A6) and (A4), we have

$$\frac{\dot{P}}{P} = \frac{\varepsilon\,V}{\xi} - r. \quad \text{(A7)}$$

From (A6) and (A7), we have:

$$\frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{P}}{P} = \varphi_m(z,m) \equiv \frac{(1-\varepsilon\,)V}{\xi} - \frac{R\,\bar{k}}{m} - \frac{T_0\,w}{m} + r. \quad (A8)$$

From (12) and (11), we have:

$$\dot{\bar{k}} = \lambda \bar{y} - \bar{k} - m - \dot{m}. \quad (A9)$$

Insert (A8) and (A5) in (A9)

$$\dot{\bar{k}} = \lambda_R \bar{k} + (1+\lambda) T_0\, w + \lambda\, m\, \frac{\dot{M}}{M} - \left(R + \frac{(1-\varepsilon)V}{\xi} \right) m, \quad (A10)$$

where

$$\lambda_R(z) \equiv \lambda\, R - 1 + R.$$

Insert (A7) and (A6) in (A10)

$$\dot{\bar{k}} = r\bar{k} + \lambda_m, \quad (A11)$$

where

$$\lambda_m(z, m) \equiv T_0\, w - R\, m - \frac{(1-\varepsilon-\lambda)V\, m}{\xi}.$$

From (11) and (6), we have:

$$T = T_0 - \frac{\sigma\, \bar{y}}{w}. \text{(A12)}$$

Insert (A12) and (A1) in (1)

$$\frac{z\,\bar{k}}{\bar{\beta}} = T_0 - \frac{\sigma\,\bar{y}}{w}, \quad (A13)$$

where we use $\bar{k} = K/\bar{N}$.Insert (A6) in (A13):

$$\bar{k} = \varphi(z, m) \equiv \left(T_0 - \frac{\sigma\,V\,m}{\xi\,w}\right)\frac{\bar{\beta}}{z}, \quad (A14)$$

where we also use (A6).Take derivatives of (A14) with respect time:

$$\dot{\bar{k}} = \frac{\partial\,\varphi}{\partial\,z}\dot{z} + \varphi_m\frac{\partial\,\varphi}{\partial\,m}, \quad (A15)$$

where we also use (A8). In (A15), we do not provide explicit expressions of $\partial\varphi/\partial z$ and$\partial\varphi/\partial m$as it is straightforward to do so but the expressions are tedious. From (A15) and (A11), we solve:

$$\dot{z} = \varphi_z(z, m) \equiv \left(r\varphi + \lambda_m - \varphi_m\frac{\partial\,\varphi}{\partial\,m}\right)\left(\frac{\partial\,\varphi}{\partial\,z}\right)^{-1}. \quad (A16)$$

Equations (A9) and (A17) are composed of two differential equations with two variables. We thus can determine $z(t)$ and $m(t)$ by (A9) and (A17). Once we determine the values of $z(t)$ and $m(t)$, we determine the rest variables by the following procedure: $r(t)$ and $w(t)$ by (A2) $\rightarrow \bar{k}(t)$ by (A15) $\rightarrow K(t) = \bar{k}(t)\bar{N}$$\rightarrow M(t)$ by (A6) $\rightarrow P(t)$ by (A8)$\rightarrow \bar{y}(t)$ by (A4)$\rightarrow F(t)$ by (A3)$\rightarrow N(t)$ by (A1)$\rightarrow T(t)$ by (1)$\rightarrow \pi(t)$ by (A8) $\rightarrow c(t),\ s(t)$, and $\bar{T}(t)$by (13) $\rightarrow \mu(t)$ by (A6)$\rightarrow \tau(t)$ by (A13). In summary, we proved the Lemma.

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