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Psychological effects and epistemological education through mathematics "abstraction" and "construction"

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Abstract

This study is part of a broader research which will be found in future work, *Psychology and epistemology of mathematical creation*, complementary work of experimental research psychology mathematics, whose investigative approach, promoting the combination type cross section paradigms and quantitative methods and qualitative and comparative method and the analytic-synthetic, based on the following idea: *to make learning as efficient, contents and methods must be appropriate to the individual particularities of the pupils, a measure of the balance between converging and diverging* dosing tasks as a promising opening to the transition from education proficiency in math performance. At this juncture, mathematical existence as ontological approach against the background of a history of "abstraction" mathematical and theoretical observations on the abstraction, realization and other mathematical thought processes, explanatory approach fulfills the context in which s mathematics constituted an important factor in psychological and methodological perspective, in a context of maximizing the educational effectiveness that depends on the quality of the methods used in teaching, focused on knowledge of the general principles of psycho-didactics not only mathematical and mental organization individual student or knowledge of the factors that make possible psycho-educational learning process.

Keywords: mathematical existence, ontological, mathematical abstraction and building, analytical insight, cognitive map, abstract entities.

JEL classification: C00, H10, O10

1 Mathematical existence as ontological problem

The theme of "mathematical existence" was a theme for reflection as the great problems of general philosophy and was established in connection with the "meaning of mathematical truths," the relationship between them and entities covered.

E.W. Beth provides an example for the *striking discrepancy* between the high degree of *certainty of mathematical truths* and world experience "any two points determine a straight line," but we do not find in our world of experience also extended the dots and dashes within the meaning of the word (Beth 1995, p. 639).

The quoted author states that "naive Platonism" mathematically original postulates the existence of points and lines in a "transcendent world" in which the human soul dwelt before the "incarnation" in a human being. After his descriptions E.W. Beth seems reminiscent of *geometric knowledge, remembrance, remembrance* of the external conditions. Marin Turlea appreciate that the conception by knowledge of mathematics, in particular the geometric rely on recall, explicitly suggests that mathematical entities are not residents of some external reality physical *world experience* (Țurlea, 2006. p.7) but you Third World as expressing K. Popper (Popper, 1974, p.87). In its conception, the "third world" is the discovery of Plato, and is divine, unchanging and true, is a "world of forms" or "ideas". Plato assess K. Popper says that this world will give us ultimate explanations (Popper, 1974, p.88) does not explain the essence, as suggested Marin Turlea (Țurlea, 2006. p.8).

Aristotelian conception, in contrast to Plato, postulates the existence of mathematical entities within the world of experience, which can be *distilled* by *abstraction*, such as E.W. Beth says (Beth, 1995, p. 640). Certainty of mathematical knowledge would explain, according to Aristotle, with the success of applications of mathematics in the natural sciences, "but this certainly faces difficulties when we consider more complex entities, which do not have corresponding examples in the *" reality of our experience "*(Turlea 2006. p.8).

Platonicist and Aristotelism constructivist conception succeeds or conceptualist, represented by Plotinus and Cusa, who declares that "mathematical entities" are constructs of human thought. As stated EW Beth, *mathematical* *knowledge is self-knowledge of human thinking* (Beth, 1995, p. 640). Surprisingly, however, after views of Marin Țurlea, it is that although Descartes has consistently emphasized the *importance of self-knowledge of human thought* and its relationship with the certainty of mathematical knowledge, he has not taken *constructivism* but *aristotelianism* (Țurlea, 2006, p.8). In fact, constructivist conception of the nature of mathematical knowledge peaked in the works of I. Kant, by exerting considerable influence on *Kantianism* general philosophy.

"The crisis foundations of mathematics" (1870 - 1900) which affected both the *analysis* and *geometry*, prompted the mathematicians to return to the question *"existence of mathematical entities"*. Once exposed details of Marin Țurlea, non-Euclidean geometries discovery shook provided in the existence of *intuitive phenomena* of mathematics. Authors like Pasch, Hilbert D. and H. Poincaré made explicit the distinction between *non-Euclidean problem* and *geometry Euclidean fundamentals*, as distinct mathematical theories, which are the competence foundational research of mathematics and geometry applicability in natural sciences, problem It belongs philosophy of natural science, after his explanation EW Beth (1995, p. 641).

Mathematicians and philosophers attention was drawn *to the fundamentals of mathematics* issue in connection with the so-called crisis of mathematics, namely the "discovery of paradoxes" logic and theory of sets. Although not the first crisis (also known at least two: the crisis ancient Pythagorean mathematics, caused by the "discovery of irrational numbers" and the crisis linked to "paradoxes of the infinitesimal calculus"), mathematics at the end of the last century, became preoccupied than ever his own foundation, the central theme is the idea of the existence of mathematical reconstruction.

It figured the problem started as *foundational programs* (logicism, formalism, intentionism) attempts to redefine the ontological status of mathematical objects and reconstruction of relevant criteria for the existence of mathematics. Only later this idea was involved in researching the relationship between mathematical objects and formal systems.

After Ilie Pārvu recording, a fundamental role in this issue have played some mathematical theorems (Gödel, Löwenheim-Skolem, Tarski) outstanding Turlea Marin called "meta-mathematical" that are related to attempts to *reduce ontological* "mathematical cutting existence of philosophical mathematical issues (Pārvu 1977, p. 67).

The literature reveals that the question whether the defendant classification of ontological status of mathematical, theoretical and methodological *infinity* theme for reflection of the human spirit ever. Then there were some lines of descent reconstruction of *mathematical ontology*, as Quine says the new trends in the general philosophy of the famous *problem of universals*, realism, conceptualism and nominalism, doctrines that will reappear in the philosophy of mathematics as representatives of *logicism, intuitionism and formalism* (Quine, 2005, p. 174).

After Quine's memoirs as a philosophy of mathematics, realism is an expression of Plato's doctrine of universal or abstract entities, as existing independently of thought. This approach assigns mathematical objects an existence itself completely autonomous, not located in space and time, independent of our conceptual and linguistic constructions.

Conceptualism declares mathematical entities as only mental constructs, creations of the human mind, out of which their existence is inconceivable. In the philosophy of mathematics version of this concept is the intuitionism (S. Poincaré, Brouwer, Weyl, Heyting). While logicism claims that entities (abstract classes) are discovered, intuitionism states that are invented, as expressed A. Fraenkel.

Nominalism support a specific position: existence is reduced to mathematical language, the finished construction signs, achievable in space and time and denies the existence of abstract entities non-spatial and non-temporal (Ţurlea, 2006. p. 10).

2 Brief History of mathematical "abstraction"

As already pointed out on other occasions, the fundamental characteristic of mathematics in Aristotle's conception, is "abstraction", in relation to which Plato in opposition net, then mathematical objects, on their existential ways, has an intermediate place between the world of ideas and the world of things accessible to the senses. After O. Becker's statements, the term "abstract" comes from genuine expression aphairesis (the abstract, get something out of a larger amount) used by him as a Latin word abstraction free translation of the Greek word. In the eighteenth century, the German texts use the term low concepts (abgezogenen Begriffen) (Becker, 1968, p. 94).

According to O. Becker, the concept of "abstracţio" has a double meaning: an operation which are set aside certain sides or traits of researched object so that it remains only a small number of features worthy of attention. This is the sense in which the term is used by Aristotle aphairesis. The alternative, of raising the general concepts, which Aristotle calls tā Koine (common), an expression applied mathematics in general terms.

For Plato, this meaning of the term "abstraction" is strongly emphasized, namely in connection with the idealization that is not too clear where Aristotle. The only time the cognitive process, which is subject to systematic treatment by Aristotle, is the operation of dissociation, the separation (chorismos) carried out by thinking about things that are, in themselves, can not be separated: "Thus says Aristotle, objects mathematical themselves are not separated, are conceived in a state of separation, thought "(Aristotle, De anima, III, 7, 431 b, 15 and following).

In Aristotle's conception, the mathematician is in opposition to physics ("natural philosopher"), but also to metaphysical ("first philosopher"). Both studies specifically object physics is concentrated on the study of the universe existence, the objects undergoing processing, and knowledge metaphysics is committed and immutable eternal essences. These areas, says Aristotle, are essences (ouisai), although independent and mathematical object acquires its existence only after the spiritual work of abstraction, carried out by the mathematician.

O. Becker believes that at Aristotle looms dint of a further orientation towards nominalism and more, an element "subjectivist". The objects appear to exist in mathematical thinking, and yet they are separated by only concrete objects in mind, and can not exist in a state dissociated itself from their carrier (Becker, 1968, p. 95).

Nominalist concept today is the widespread belief on the specific element essence and not only mathematically most scientists researching the phenomena of nature, but even many mathematicians, particularly in Anglo-Saxon world.

Bertrand Russell, for example, quickly abandoned Platonic position, initially intended to adopt a legally oriented empiricist sense, although later will no longer maintain the general negative attitude to admitting any a priori concept. Own empirical reasons can be highlighted in this guidance Anglo-Saxon tradition from English philosophy, such as epistemology of D. Hume. From a certain perspective this trend is offset by formalistic empiricist, when the logic of mathematics (called "logistics") has gained ground within mathematics itself.

There are two basic examples of ancient authors which served to illustrate "the general science of mathematics." First, a group of "fundamental sentences" about equality and inequality, for example: "And if the equal subtracted equal, the remaining ones are equal". This includes the consideration of Euclid Enno KOIN.

Usually, Greek term is translated by the expression "common notions"; but as opines O. Becker, remains undecided whether it is the representation of objects of general or specific concepts spread among people, or both (Becker, 1968, p. 96). Secondly, a group of sentences from the general theory of propositions, as it was established by Eudoxiu of Cnidos and formulation appears in Book V of Euclid's Elements. In ancient mathematics have great significance both groups of sentences, and the sentences operations.

Aristotle notes that before theorem about the possibility of changing between them Medes and extreme-was demonstrated separately for numbers, segments and intervals bodies. Now, however, continues Aristotle, making an allusion to the theory's propositions Eudoxiu the demonstration is generally performed once for all kinds of sizes, because the relationship in question is no longer confined to segments and bodies as such, but to "what is taken as a whole (Katholou) (Aristotle, raw analytic, I, 5, 74, 17-25).

G. Becker emphasizes that in this case, the term katholou a generalization does not mean in the sense of climbing higher order species, but it is used in the sense of a true formalization (Becker, 1968, p. 98). In Metaphysics, Aristotle shows that the mathematician refers only to what has existence by abstraction (conscious omission), taking into account the different categories objects (in terms of their shape, such as points, bodies, lines, areas) only terms of their quantitative side of their character continuously and nothing but them (Aristotle, Metaphysics, MK3, 1061 a 28-b6).

After O. Becker's view it is clear that "abstract mathematical concept" lacks independence ("substantiality") mathematical objects. Despite this difference exists, however, a general affinity between mathematics and general ontology, because of both formal, which he later contributed to the formation of a Mathesis idea universalis. Neo-Platonists reveal obscure character of this general mathematics and considered it as a form superior spiritual science's remarks Proclus to Euclid's Elements is an example.

Starting with Viète and Descartes, a new form of mathematics, "free of the geometric shape" and "limitation imposed by using particular numerical values," as O. Becker would say.

The big mathematical contribution made a positive nature of Descartes, is "analytic geometry foundation ... the first application of this Mathesis universalis to a particular area, reached in fact only with Leibniz. While Descartes remains a "geometric analytic" Leibniz appears as "arithmetician or universal algoritmician".

Leibniz introduced "differentials" (dx, dy, dt etc.), but he also discovered the concept of "determining" where we find the first step towards a theory of invariants "(Becker, 1968, p.100). Also his name links the concept of "function" and the main interpretation of the concept of "representation", a term that has a double meaning: a representation of an object in a picture or, conversely, to achieve a concept through an object real, of a model. How deep is intertwined with the philosophy of mathematics in the thought of Leibniz, his example shows us tied to monads as a mirror of the universe.

Relationship between the divine world and the world of human thought, limited and lacking in clarity, it is illustrated with a mathematical comparison: as hyperbole extending to infinity can be represented by a central projection cone determined geometric generator on a ellipse, so the curve lies entirely in the infinite, as we can imagine divine and infinite universe of ideas, designed the finite world of human representations. As curved mirrors the distorted images restore the original object, as it reflects the macrocosm of divine human monads.

Leibniz's contribution does not stop there: he develops a logical calculus, calculus called universalis, with an important theorem proving not only data but also the invention of new sentences or abstract symbols shifting balance "intuitive". The calculation logic is to serve to eliminate errors of judgment but also to new insights. O. Becker said that "he was thinking deductive models ideal for systems that could be applied to any possible material" (Becker, 1968, p.102).

The new mathematics had to have a rigorous deductive character and had to be written and expressed using precise graphic signs of a "characteristic universalis", with two main branches: "logistics" and "combinatorial feature" a symbolic mathematical quantity and quality.

O. Becker considers that Leibniz introduced before all logical calculations of various types, based on a concept so intensive - elementary calculation of predicates or calculation properties - as well as a broad concept - algebra classes or crowds. In 1690, Leibniz gave a formal abstract representation, conscious, calculated or designed as a theory "of the man who contains and what is contained" (the conscious et conteno). It seems that he would have provided the first theory of Boolean abstract structures, about 150 years before G. Boole.

In the eighteenth century, differential and integral calculus operate the system, variational calculation and the first beginnings of a theory of ordinary differential equations with partial derivatives. Differential geometry also makes progress and appears in the nineteenth century theory of functions of a complex variable theory of elliptic functions and other functions of higher type; appear projective geometry and other subjects that go beyond the space geometric intuitive geometry "Euclidean" as to give rise to a general theory of invariants to groups of transformations in n-dimensional variety. It develops group theory, number theory on new foundations, the theory of solving algebraic equations by radicals, turning into abstract algebra discipline, which contains numerical algebra classical era as a particular case.

In terms of philosophy, presents relevance the theory developed by G. Cantor, which removes Aristotelian thesis of the nature of the infinite potential, including acceptable sentence B. Bolzano, who created the theory and actual infinity, which underlies many mathematical disciplines special. It also develops the theory of "structures", closely linked to the formal structure of mathematical logic, which plays an important role so-called "distributive networks" (Becker, 1968, p.96).

In conclusion, we can say that, seen from the point of view of its own structure, Greek mathematics differs from the preceding eras of the ancient Orient through a conscious acceptance of the infinite. The infinite which philosophy developed, is Aristotle, who saw its essence opportunity to continue without interruption a process. In his conception, the infinite can not exist otherwise than in pure possibility "in potency". In contrast to Plato, Aristotle believes that the existence of mathematical abstraction is based, which means that he does not conceive as mathematical figures in and of itself made a substantial essence, but sees in them items that appeared in physical bodies through concrete -a process of abstraction (aphairesis). He thinks that the objects of thought, products of human or divine spirit.

As O. Becker stated , Stagiritul conceive essence of mathematical objects 'general' in dependency theory of abstraction, for example, "propositions" as Eudoxiu's theory, they regard as a higher stage of abstraction, which has no regular character a generalization operations, but the specific nature of a formalization.

Before succinct analysis systems ontology formal ontology of abstract objects and psychological effects of abstraction, we propose some theoretical observations on abstraction and realization, as well as other mathematical thought processes are generalization, restriction, construction, analysis, symbolization, interpretation.

3 Comments on theoretical abstraction, realization and other mathematical thought processes

However much we insist on psychology and pedagogy, we can reach the expected results, psycho-pedagogical, without understanding the higher cognitive processes, enhanced especially in the logical-mathematical understanding.

Abstraction says. ZP Dienes, consists of "extracting what is common to a number of different situations and the removal of what we decide that we must consider" jitter "or "irrelevance" (Dienes 1973, p. 57). This explanation uses the term "jitter" (the original noise = noise) in the figurative sense widest applicable to all phenomena or disturbing elements, by their mere presence, making a particular process or finding some way to correct solution problem. The jamming can be caused either by information, methodical procedures, or some ideas, tendencies, impulses arising in that person, skills or formal stereotypes.

In line with this concept, abstraction is a "synonym for training classes," its finality consisting of notification attributes on which elements may or may not be elected as members of a class. Thus, every word that denotes a "type of things" means a class. We appreciate, in agreement with ZP Dienes that necessarily attributes that an object still possess before being appointed chair, say, are not always appreciated conscious and therein lies one of the differences between mathematical thinking and habitual thinking. The conclusion drawn by ZP Dienes is clear: "to become a math class operant consciousness is almost indispensable '(Dienes 1973, p. 58).

Following his reasoning ZP Dienes make the following clarifications: by abstracting complex mathematical classify objects, as these objects of thought: the practice can not reach the "sense" of these items and decide properly if a complex is also some other complex structures; mathematicians insights, but they do not rely only on them and therefore, in our case, the identity structures must be clearly demonstrated. In this regard, the author gives the following example: not self-evident that the overlap of two transformations of type:

$$X = ax + by \qquad\qquad u = cX + dY$$

and

$$Y = -bx + ay \qquad\qquad v = -dX + cY$$

It will combine exactly the same as multiplying the complex:

$(a + bi) \times (c + di)$, in which $(i^2 = -1)$.

Intuition may tell us that things are so, but must be demonstrated. The proof is accomplished by carrying out the substitution in the first set of processing from which is obtained:

$u = (ac - bd)x + (bc + ad)y$
$v = -(bc + ad)x + (ac-bd) y$

and if it performs complex multiplication is reached:

$(ac - bd) + (bc - ad)i$.

If it is determined following correspondence:

Top left coefficient matrix corresponds to the "real" and the coefficient matrix corresponds to the upper right of the party "imaginary" can "translate" activity in the other, since both have the same properties, to the extent that it's inter- their property relations.

In this way they identified two mathematical structures that actually looks different from one another and then found that, from a certain point of view, they have the same structure. In this context, interpretations of what constitutes i2 = -1 what Z.P. Dienes called "jitter". If you eliminate this interference is noticeable series of attributes that must possess a structure to be described as "complex algebraic expression."

ZP Dienes notes that there is in fact isomorphism abstraction, which is more than a two-way correspondence; it means that, to the extent that it's at least an operation, if operating with B on C's to get the result in one instance, then operating with the isomorphic isomorphic to B of A we obtain as a result, isomorphic C's in the other case.

In this case you can usually tell that the two situations are isomorphic in terms of operations applied, implying a perfect correlation - one exception being sufficient to refute isomorphism. "The formation of isomorphisms, continues ZP Dienes, is the process by which we come to abstractions" who will "become some complex class defining properties for the scope of which coincides with that abstraction and whose elements are all possible structures with specific properties the same complex ". Formation of abstractions "clarified things" and so we know where we are. Formation of abstractions is an end point in a cycle both psychologically and in one mathematically.

The reverse is fleshing abstraction, understood as evidence of a structural concrete example, if the class was not made "by building in her element", but by logical combinations of attributes, which may become necessary in cases of existence theorems.

A theorem of existence can be demonstrated in at least two cases: one can ask whether the combination of attributes considered is essentially impossible due to the nature of these attributes (in which case we must decide whether we have any grounds logic lead us to conclude that things are not defined by these attributes); The second question would be that we could simply ask how it looks and how it behaves in reality, things we are talking about.

In conclusion, we can say that abstraction is to move from classroom items and more concrete, to move from the classroom to the elements. Isomorphisms help us to establish with certainty what elements belong to a certain class.

As Jerome Bruner was to demonstrate the formation of an abstract idea in the sense of forming a class is actually a concept irreversible psychological point of view, it is therefore impossible to regain the "pre-conceptual innocence" after I made a class, we can return to its elements, but things will never be as before.

Materialization serves planning routes to be followed by subsequent builders of the class, but will not turn thought processes of those who started to build that class. In comparison, generalization is a reversible process, turning shrinking or customization, since we are dealing with previously formulated classes.

In the narrow sense, generalization is to become aware of a report for inclusion in several classes and a class in another. It means discovering that if x belongs to the class A, he invariably belongs and class B. So, is included in Class A Class B or say, in fact, may be extended or class within the class generalized B. Mathematicians believe that to produce a mathematical generalization, "should take place in advance a certain abstraction" (Dienes 1973, p. 60).

Usually, generalization occurs when existing situations are easily extended over some imaginary circumstances have not served as an example, but are essentially the same type. O. Becker shows that when there is awareness that a certain structure of rules extend to a larger crowds examples than expected, then it can be symbolized in this general form. But symbolism does not work unless it can be stated that certain real situations are expressed to him. Between generalization and symbolization must achieve a two-way movement.

Symbolism can be or not an abstraction and symbolization reverse process is what is called interpretation. If the symbolism is reached as a means of communication of common properties of different types of situations, Becker A, with the same structure shows that it "symbolizes an abstract symbolism". In reality it may symbolize a generalization and, in this case, "symbols are only used as a shorthand native language of the subject, to describe experiences still unconnected" (Dienes 1973, p. 60).

The condition that generalization to become a functional part of the thinking of an individual, it must reach to easily execute and the reverse of, "customize" or "restriction".

In mathematics, generalization can also be made formal. Substitution of formal class elements restricted earlier summarizes the formal customization. For generalizations and customizations to be considered mathematical thinking, that they must be interpreted in concrete situations and relevant. The restriction is specific mathematical thinking because it operates in mathematics classes we associate with other classes, uniting them. Attribute A and attribute B is to apply the new class and we are dealing with a restrictive procedure, which runs opposite of generalization.

Brief considerations mentioned above have generalized the concept of ZP Dienes, a type of cognitive organization of learning mathematics that explains the process of abstraction - systematization.

From its chart, it is inferred that abstraction is regarded as a potential activity consisting in being able to go from one element to another class. According to the author, this ability becomes, by exercise, a kind of mental attitude towards "all things" that are likely to be classed together. It is possible to generalize several variables at once, which means that some of the statements generalization may be issued and vice versa is possible to customize several variables at once.

It notes that ZP Dienes did not include logical analysis, arguing that when making a generalization ways "logical" lead to what he calls "analytical insights." He believes "cognitive map" as a "cell integrated into an organic learning process there are cells with different structures, each structure as tailor specific type of learning which contributes to the achievement" (Dienes 1973, p. 60).

Dienes them find this model more suitable for developing a theory of learning than stimulus-response model.

The various methods shown in "cognitive map" meet in a certain way to play thread mathematical thinking training model. These processes can not be considered in isolation, but in relation to a methodological suggestions on learning activities and training strategies, designed to help, winning learning experiences, training metacognitive structures, patterns of internalization desirable behavior"(Bunăiaşu, CM, 2011, p. 149).

The author of "cognitive map" believes that there are many other variables, both psychological and mathematical, which will complete this picture and only then could give a coherent explanation about psychological processes and logical when we think mathematically. Our image will be much clearer if we bring into question in future issues of ontology ontology formal systems and abstract objects, in order to elucidate the importance of using the mathematical model in forming a cognitive behavior relevant to a modern educational-training process.

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